Estimation of the Percentile Maximum Time Interval Error of Gaussian White Phase Noise

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Abstract

The MTIE is historically one of the main time-domain quantities considered for the specification of clock stability requirements in telecommunications standards. In this paper, MTIE is first introduced according to its classical definition and as a percentile quantity. Then, the percentile MTIE is estimated, under the assumption of Time Error (TE) affected by Gaussian White Phase Modulation noise, by deriving the probability distribution of the TE spanned range as a function of the noise standard deviation σ. The formulas herein derived may allow to interpret common Allan-variance factory specifications in terms of percentile MTIE as well. In order to support the theory with sound experimental evidence, some results measured on state-of-the-art telecommunications clocks are eventually provided.

1. Introduction

Network synchronization [1]—[3] has become a hot topic in international standard bodies in the last years, since the introduction of the circuit-switched data networks and of new advanced digital services yielded the need of more stringent synchronization requirements, in order to avoid slips in the equipment input elastic stores [4][5]. Moreover, the ongoing spreading of Synchronous Digital Hierarchy (SDH) [6] and Synchronous Optical NETwork (SONET) technology in transmission networks has further enhanced the need of network synchronization, as SDH transmission takes advantage from it (and may rely on it).

A major topic of discussion in standard bodies is clock stability characterization. Among the quantities considered for the specification of stability requirements [7][8], the Maximum Time Interval Error (MTIE) has played historically a major role for characterizing time and frequency performance in digital telecommunications networks [9]—[11], as specifications in terms of MTIE are well suited to support the design of equipment buffer size.

Unlike other frequency stability measures [12]—[15] (such as the Allan variance), all implying some kind of data averaging, MTIE is a rough *peak* measure of the time deviation of a clock with respect to a known reference. This involves some delicate issues in accomplishing its practical measurement and in its conceptual analysis [16][17]. In order to cope with these issues, therefore, recently ITU-T [7] redefined MTIE as a percentile quantity, i.e. a peak level which is not exceeded in more than, say, 99% of the cases.

In this paper, MTIE is first introduced according to its classical definition and as a percentile quantity. Then, the percentile MTIE is estimated, under the assumption of Time Error (TE) affected by Gaussian White Phase Modulation (WPM) noise, by deriving the probability distribution of the TE spanned range as a function of the noise standard deviation σ. Since most oscillators are often specified by the manufacturer only in terms of their noise power spectrum or Allan variance, the formulas herein derived may allow to interpret such stability information in terms of percentile MTIE as well. In order to support the theoretical treatment with experimental evidence, some results measured on state-of-the-art telecommunications clocks are eventually provided.

2. Definition of MTIE

A thorough treatment of MTIE and of its properties can be found in [16]. Here, solely the main definitions are summarized for the sake of understanding and to provide the reader with the background concepts.

A general expression describing a pseudo-periodic waveform which models the timing signal $s(t)$ at the output of clocks is given by [12][13]

$$s(t) = A \sin \Phi(t) \tag{1}$$

where A is the peak amplitude and $\Phi(t)$ is the *total instantaneous phase*, expressing the ideal linear phase increasing with t and any frequency drift or random phase fluctuation.

The generated *Time* function $\mathrm{T}(t)$ of a clock is defined, in terms of its total instantaneous phase, as

$$\mathrm{T}(t) = \frac{\Phi(t)}{2\pi\nu_{\mathrm{nom}}} \tag{2}$$

where ν_{nom} represents the oscillator nominal frequency. It is worthwhile noticing that for an ideal clock $\mathrm{T}_{\mathrm{id}}(t)=t$ holds, as expected. For a given clock, the *Time Error* function $\mathrm{TE}(t)$ (in standards also called $x(t)$) between its time $\mathrm{T}(t)$ and a reference time $\mathrm{T}_{\mathrm{ref}}(t)$ is defined as

$$x(t) \equiv \mathrm{TE}(t) = \mathrm{T}(t)\text{-}\mathrm{T}_{\mathrm{ref}}(t) \tag{3}$$

The *Maximum Time Interval Error* function $\mathrm{MTIE}(\tau,T)$ is the maximum peak-to-peak variation of TE in all the possible observation intervals τ (in former standards [9][10] denoted also as S) within a measurement period T (see Fig. 1) and is defined as

$$\mathrm{MTIE}(\tau,T) = \max_{0\le t_0 \le T-\tau} \left\{ \max_{t_0 \le t \le t_0+\tau} \left[\mathrm{TE}(t)\right] - \min_{t_0 \le t \le t_0+\tau} \left[\mathrm{TE}(t)\right] \right\} \tag{4}$$

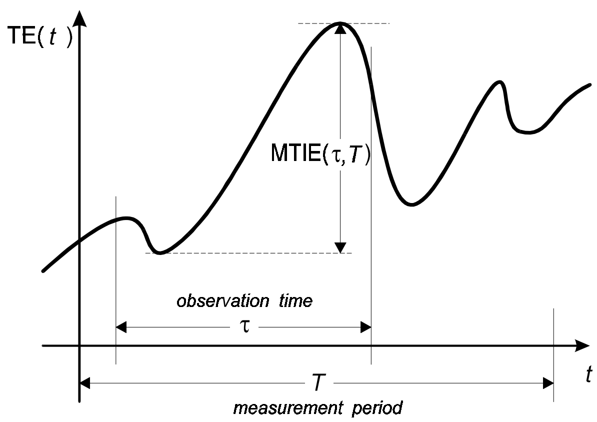

Fig. 1: Definition of MTIE(τ,*T*)

It should be noted, however, that the standards in force specify the MTIE limits simply as a function of τ (or *S*), thus implicitly assuming

$$\mathrm{MTIE}(\tau) = \lim_{T\to\infty} \mathrm{MTIE}(\tau,T) \tag{5}$$

Now, if phase fluctuations are modelled by a Gaussian probability distribution of the amplitudes (this assumption has been verified experimentally, especially under WPM noise [18]), the maximum range spanned by TE may reach infinitely large values. Increasing the measurement period T allows to observe the tails of the distribution: they are less likely but, in principle, unlimited.

For this reason, not specifying T (i.e. simply assuming it "large enough") makes MTIE(τ) ambiguous. On the other hand, if a particular value of T is specified, the measured value MTIE(τ,T) depends in general not only on τ but also, to a smaller extent, on the overall period T during which the clock has been under test [19], since limiting T implies a low-frequency cut-off on the clock signal. This yields some delicate issues in settling the measurement procedure, as broadly discussed in [16]. Moreover, a measurement of MTIE(τ,T) based on a single measurement period T depends on the particular realization of the TE process and therefore does not contribute to a rigorous characterization of the oscillator under test. For all these different reasons, the need has been felt to probe further and to modify the MTIE definition so that to maintain its peak information but to overcome some ambiguities in its original definition.

In order to cope with the reported issues, recently ITU-T [7] redefined MTIE(τ,T) as a specified *percentile* β of the random variable (cf. eq. (4))

$$X = \max_{0\le t_0\le T-\tau}\left\{\max_{t_0\le t\le t_0+\tau}\left[\mathrm{TE}(t)\right]-\min_{t_0\le t\le t_0+\tau}\left[\mathrm{TE}(t)\right]\right\} \qquad (6).$$

In other words, based on such a definition, a MTIE β-percentile mask

$$\mathrm{MTIE}_{\beta-\mathrm{perc}}(\tau)\le a \qquad (7)$$

gives the limit a not to be exceeded in more than, say, β=99% of the cases.

This definition of the percentile MTIE seems a meaningful tool to specify the required phase stability of clocks, featuring in fact a few advantages. In the next section, the issue of estimating the percentile MTIE basing on some statistical description of the underlying noise is faced by studying the statistical properties of the peak-to-peak phase fluctuations.

3. Estimating the MTIE$_{\mathrm{perc}}$ of Gaussian WPM Noise

The percentile MTIE is herein estimated in the case of TE(t) affected by Gaussian WPM noise basing on the knowledge of the noise standard deviation σ, by deriving the probability distribution of the TE(t) spanned range, in order to allow a first assessment of the clock stability in terms of MTIE on the basis of e.g. Allan-variance factory specifications [17].

The assumption of Gaussian noise amplitude distribution has been supported by experimental evidence [18] and may be interpreted as the combined result of many different independent perturbations acting on the clock signal. The case of WPM noise, on the other hand, is the simplest to be treated and it is quite important when considering telecommunications clocks, that often exhibit a predominant WPM noise, due for example to digital loop algorithms, in a wide range of τ values.

3.1. The Range Distribution

The study of the range spanned by a random signal dates back to the beginning of the century, applied to biometrics studies as well as to the statistical control of industrial processes [20]—[22].

Recalling the MTIE definition (4), let us extract the central part defining the range $Z(\tau)$ spanned by the peak-to-peak phase fluctuations over an observation interval τ

$$Z(\tau)=\left\{\max_{t_0\le t\le t_0+\tau}\left[\mathrm{TE}(t)\right]-\min_{t_0\le t\le t_0+\tau}\left[\mathrm{TE}(t)\right]\right\}_{\forall t_0} \qquad (8)$$

where the initial instant t_0 can be whatever and $Z(\tau)$ is a random variable. During the observation interval τ the time error function TE(t) has a continuos evolution, but without loss of generality it may be seen as a series of instantaneous values x_i (result of any measurement procedure) spaced of a sampling time τ_0. The range Z is thus the peak-to-peak excursion spanned by n+1 samples of the process TE(t), where $\tau=n\tau_0$. In the case of Gaussian WPM noise, each sample x_i of the process TE(t) is normally distributed and is independent from all the other samples (whiteness assumption). Starting from this basis it is now possible to infer the probability distribution and the percentile values of the range Z.

By the assumption of zero-mean Gaussian process [18], the probability density function $f_{x_i}(x)$=Prob($x<x_i\le x+\mathrm{d}x$)/dx results

$$f_{x_i}(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{x^2}{2\sigma^2}} \qquad (9)$$

where σ is the distribution standard deviation. Being the samples x_i independent, the probability density function of the maximum value x_{MAX} among n+1 samples, i.e. $g_{x_{\mathrm{MAX}}}(w)$=Prob($w<x_{\mathrm{MAX}}\le w+\mathrm{d}w$)/d$w$, is given by

$$g_{x_{\mathrm{MAX}}}(w)=(n+1)[F_{x_i}(w)]^n f_{x_i}(w)dw \qquad (10)$$

where $F_{x_i}(w)$ *is* the cumulative distribution function of x_i, i.e. in this case

$$F_{x_i}(w)=\int_{-\infty}^{w} f_{x_i}(x)\mathrm{d}x=\frac{1}{2}\left(1+Erf\left(\frac{w}{\sqrt{2}\sigma}\right)\right) \qquad (11)$$

and $Erf(y)$ is the Error Function

$$Erf(y)=\frac{2}{\sqrt{\pi}}\int_0^y e^{-t^2}\mathrm{d}t \qquad (12).$$

An analogous distribution holds for the minimum value x_{MIN}. By considering the joint probability distribution (x_{MAX}, x_{MIN}) and the definition (8) of the range Z, the *range probability density function* $\phi_Z(z)$=Prob($z<Z\le z$+dz)/dz is finally found as

$$\phi_Z(z)=\frac{(n+1)n}{2\pi\sigma^2}\frac{1}{2^{n-1}}\cdot \int_{-\infty}^{\infty} e^{-\frac{(z+y)^2}{2\sigma^2}}e^{-\frac{y^2}{2\sigma^2}}\left[Erf\left(\frac{z+y}{\sqrt{2}\sigma}\right)-Erf\left(\frac{y}{\sqrt{2}\sigma}\right)\right]^{n-1}\mathrm{d}y \qquad (13)$$

and can be evaluated, for a given n, by means of some numerical integrations.

In Fig. 2 the range probability density function $\phi_Z(z)$ is plotted for some different values of n and having normalized z to σ. We note that, as n increases, the observation of a larger range becomes more likely.

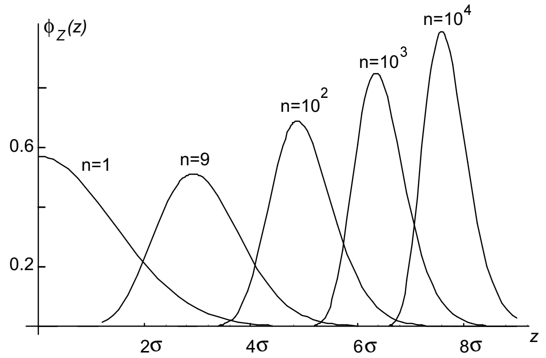

Fig. 2: The range probability density function $\phi_Z(z)$

3.2. Estimating the Percentile MTIE

From the range probability distribution derived, it is now possible to estimate the β-percentile a, i.e. the threshold level a which is expected to be exceeded only in a 1-β percentage of times, as

$$\Phi_Z(a)\equiv\int_0^a \phi_Z(z)\mathrm{d}z=\beta \qquad (14).$$

The threshold level a, dependent on n and β, represents the $MTIE_{\beta\text{-perc}}(\tau)$ (cf. eq. (7)). Therefore, by the evaluation of eq. (14) for different values of n and β, the percentile MTIE can be estimated for different observation times $\tau=n\tau_0$. Examples of threshold values $MTIE_{\beta\text{-perc}}(\tau)$ so estimated are plotted in Fig. 3, as multiple of the standard deviation σ of the parental Gaussian distribution, versus n and for β= 0.97, 0.99, 0.999.

To the reader interested in numerical computing, it is worth to mention that solving eq. (14) can be simplified by expressing $\Phi_Z(a)$ as follows [22]

$$\Phi_Z(a) = \int_{-\infty}^{\infty} (n+1) f_{x_i}(x) \left[F_{x_i}(x+a) - F_{x_i}(x) \right]^n \mathrm{d}x \qquad (15)$$

where $f_{x_i}(x)$ and $F_{x_i}(x)$ are the functions defined by (9) and (11) respectively.

In conclusion, we note that estimating the percentile MTIE is then possible, under the assumption of Gaussian WPM noise, by the only knowledge of the standard deviation σ of the underlying noise. Moreover, it is now worthwhile noticing that, under this assumption, σ is directly linked for example to the value of the Allan deviation $\sigma_y(\tau_0)$ [23], thus enabling a first assessment of the percentile MTIE basing on common Allan-variance factory specifications [17].

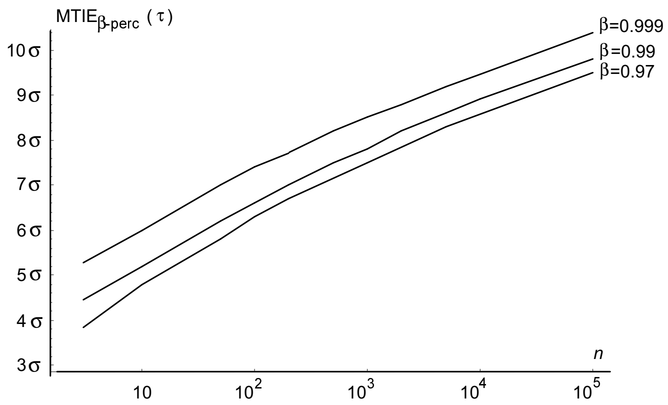

Fig. 3: Examples of threshold values $MTIE_{\beta\text{-perc}}(\tau)$ estimated

4. Experimental Work

In order to support the theory treated above with experimental evidence, some measurement results are eventually provided in this section. These results were chosen among those measured throughout the last few years by testing telecommunications clocks of different types, and they are sound examples of how the theoretical analysis of Sec. 3. may apply to practice, where also other kinds of noise may be revealed together with Gaussian WPM noise.

4.1. Measurement Technique

Measurements were accomplished in the standard *synchronized clocks configuration* [7][8][15] and, according to an usual procedure (detailed for MTIE in [16]), were based on the time-domain measurement of the TE process *x(t)* between the output of the slave Clock Under Test (CUT) and its input reference (i.e. the quantities $T(t)$ and $T_{ref}(t)$, respectively, in eq. (3)).

The measurement test bench is then outlined in Fig. 4. A high-performance time counter, with a resolution of 200 ps, measured the TE between the timing signals (both 2.048 MHz G.703/§10 [24]). The reference was synthesized from a rubidium frequency standard which also supplied the time base to the time counter. Finally, the time counter was driven via a GPIB IEEE488.2 interface by a laptop computer which managed data acquisition, processing and visualization.

Sequences of N TE samples $\{x_i\}$, defined as

$$x_i = x(t_0 + (i-1)\tau_0) \quad i = 1, 2, \ldots, N \qquad (16)$$

where t_0 is the initial observation time and τ_0 is the sampling period, were measured and stored for numerical post-processing over a total measurement interval $T=(N-1)\tau_0$. Starting from such TE measured sequences $\{x_i\}$, the stability quantities *Allan Deviation* ADEV(τ), *Modified Allan Deviation* MADEV(τ) and the classical MTIE(τ) were then evaluated by computing the standard estimators specified by ITU-T [7] and ETSI [8].

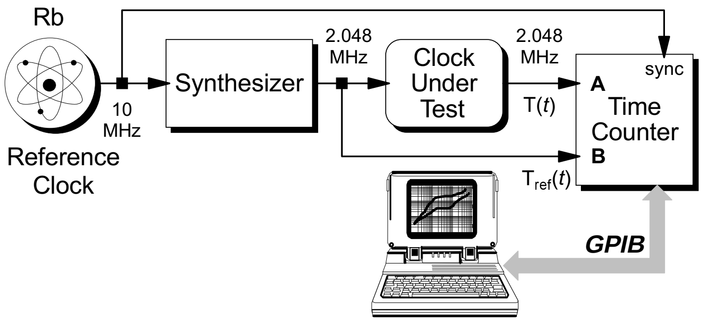

Fig. 4: Measurement test bench

4.2. Estimating the Percentile MTIE in Practice

According to the theoretical analysis of Sec. 3., percentile MTIE values can be estimated basing on the standard deviation σ of the underlying noise, under the assumption of Gaussian WPM noise. Under this hypothesis, σ can be evaluated [23] as proportional to the Allan variance; for example, for $\tau=\tau_0$, as

$$\sigma = \frac{\tau_0}{\sqrt{3}} \sigma_y(\tau_0) \qquad (17).$$

Then, by solving eq. (14) or by inspection of Fig. 3, $MTIE_{\beta\text{-perc}}(\tau)$ values can be inferred as multiples of σ. For example, with β=0.99 and for $\tau=10^5\tau_0$, a $MTIE_{99\%}$ value of about 9.5σ is estimated.

Simulative and experimental work showed that the above formulas for estimating the percentile MTIE work fine if the underlying noise can be well modelled as a Gaussian WPM noise in the τ range of interest (cf. the dots marked as "estimate" or "estimate 1" in the graphs shown). On the other hand, in the range of τ where additional slower noises are not negligible compared to the WPM noise, like in most measurement results shown in this paper, then the percentile MTIE values inferred get underestimated.

However, in order to attempt a better estimate with this model but by taking into account other noise components too, it is still possible to evaluate the overall standard deviation σ of the measured TE sequence $\{x_i\}$ and then approximate the noise model as Gaussian WPM (the σ so evaluated, obviously, is not the same as calculated through the (17), which takes into account only the WPM noise component). Moreover, since in the case of measurement in the synchronized clocks configuration the ADEV curve exhibits the WPM slope for τ greater than the loop time constant [15], regardless of the kind of noise generated by the internal slaved oscillator, we can think also to apply the (17) to the $\sigma_y(\tau_0)$ value obtainable by extrapolating such trend back to $\tau=\tau_0$, so treating it as a true WPM noise. Both the aforementioned techniques were applied to the measurement results shown in this paper to produce additional estimates (cf. the dots marked as "estimate 2" in the graphs shown).

4.3. Measurement Results

The measurement results provided in Figs. 5a-5b through 7a-7b were obtained by testing two state-of-the-art stand-alone slave clocks for synchronization networks (suppliers A and B) and the clock of a SDH Digital Cross-Connect (DXC) 4/3/1 (supplier C). With ITU-T standard terminology, we refer to the first two as Stand-Alone Synchronization Equipment (SASE) and to the last one as SDH Equipment Clock (SEC). The SASEs A and B were equipped with a quartz Oven Controlled Crystal Oscillator (OCXO), the SEC C with a quartz Temperature Compensated Crystal Oscillator (TCXO). Moreover, also the results of the test-bench background noise are provided in Figs. 8a-8b; they were measured by splitting and directly feeding the reference timing signal into the time counter input ports.

Figs. 5a, 6a, 7a and 8a depict the ADEV(τ) and MADEV(τ) curves measured according to the procedure outlined in Sec. 4.1. From their trends, the noise characteristics of the CUTs can be inferred and summarized as follows. The SASE A (Fig. 5a) exhibits a WPM noise for τ<5 s, while for τ>50 s the clock is inside the loop control bandwidth (measurement in the synchronized clocks configuration) and thus exhibits the typical residual noise whose ADEV is proportional to τ^{-1} as for pure WPM noise [15]. The SASE B (Fig. 6a) exhibits a similar behaviour: WPM noise for τ<1 s, White Frequency Modulation (WFM) noise for 1 s<τ<300 s, while for τ>300 s the clock is inside the loop control bandwidth. The SEC C (Fig. 7a) exhibits WPM noise for τ<1 s, while for τ>5 s the clock is inside the loop control bandwidth; moreover, a superposed sinusoidal phase modulation having period of about 10 s is revealed by the bump at τ=4 s and the ripple on the ADEV and MADEV curves. Finally, the test-bench background noise (Fig. 8a), as expected, exhibits a clean WPM noise in all the measurement range of τ.

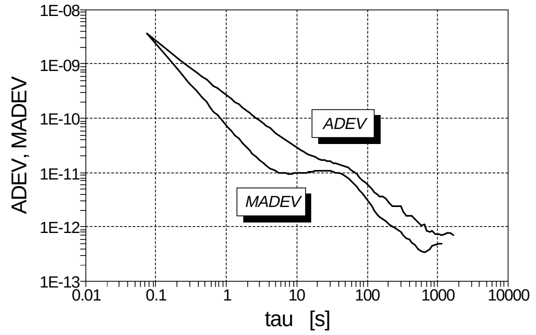

Fig. 5a: ADEV(τ) and MADEV(τ) curves measured on the SASE A (N=48350, $\tau_0\cong$75 ms, T=3600 s)

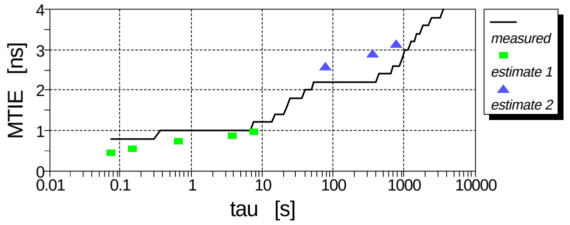

Fig. 5b: MTIE(τ) curve measured on the SASE A (N=48350, $\tau_0\cong$75 ms, T=3600 s) and $MTIE_{99\%}$ estimates

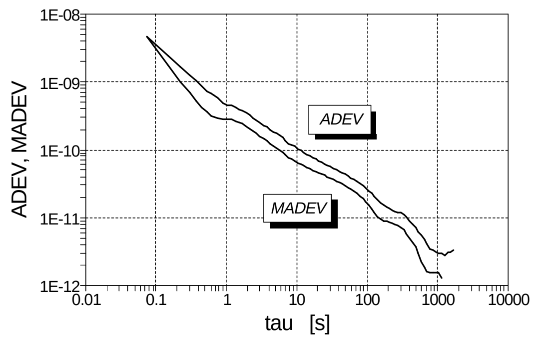

Fig. 6a: ADEV(τ) and MADEV(τ) curves measured on the SASE B (N=47850, $\tau_0\cong$75 ms, T=3600 s)

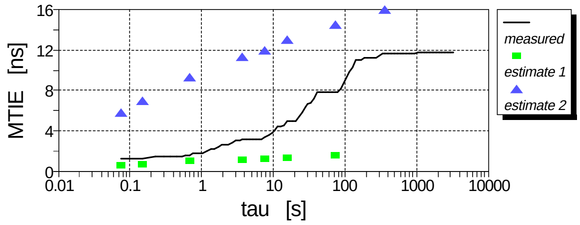

Fig. 6b: MTIE(τ) curve measured on the SASE B (N=47850, $\tau_0\cong$75 ms, T=3600 s) and $MTIE_{99\%}$ estimates

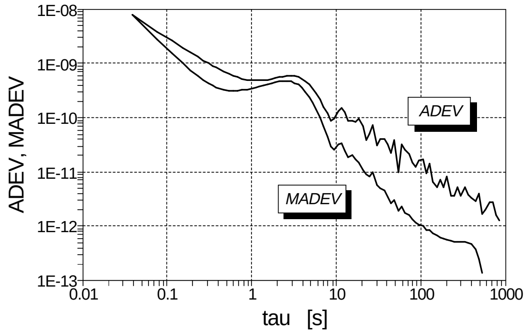

Fig. 7a: ADEV(τ) and MADEV(τ) curves measured on the SEC C (N=46000, $\tau_0\cong$38 ms, T=1755 s)

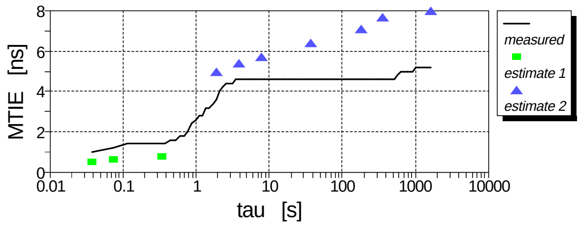

Fig. 7b: MTIE(τ) curve measured on the SEC C (N=46000, $\tau_0\cong$38 ms, T=1755 s) and $MTIE_{99\%}$ estimates

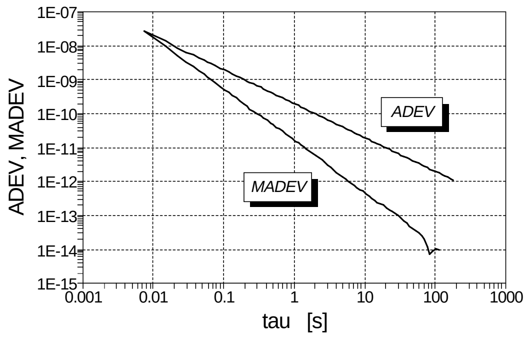

Fig. 8a: ADEV(τ) and MADEV(τ) curves measured on the test-bench background noise (N=50000, $\tau_0\cong$7.5 ms, T=375 s)

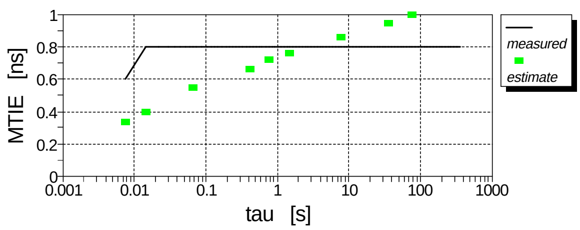

Fig. 8b: MTIE(τ) curve measured on the test-bench background noise (N=50000, $\tau_0\cong$7.5 ms, T=375 s) and $MTIE_{99\%}$ estimates

On the other hand, Figs. 5b, 6b, 7b and 8b depict the classical MTIE(τ) curves (solid line) measured according to the procedure outlined in Sec. 4.1. and compared to the percentile $MTIE_{99\%}$ values (filled squares and triangles) estimated according to Sec. 4.2. In all the graphs shown, the squares marked as "estimate" or "estimate 1" represent values estimated basing on the σ evaluated as the (17). Outside the τ range where the Gaussian WPM assumption holds, the "estimate 2" triangles represent an improved estimate according to what stated in Sec. 4.2.

In particular, in Fig. 5b (SASE A), the "estimate 2" values have been obtained by extrapolating in Fig. 5a the ADEV section for τ>50 s back to $\tau=\tau_0$ to get a new value of σ. In Fig. 6b (SASE B), the "estimate 1" values for τ>1 s appear heavily underestimated due to the predominant WFM noise which makes the Gaussian WPM assumption fail; the "estimate 2" values, on the other hand, have been obtained by evaluating the overall standard deviation σ of the measured TE sequence $\{x_i\}$ and then estimating the percentile $MTIE_{99\%}$ values from that value. The same holds for the "estimate 2" values plotted in Fig. 7b (SEC C). For the results depicted in Fig. 8b, finally, due to the pure WPM noise revealed, the standard deviation σ evaluated from $\{x_i\}$ has the same value as that evaluated as (17); only the first estimate was then plotted.

It is worthwhile recalling, in this last case, that the test-bench background noise is mainly due to the trigger and quantization errors of the digital time counter. It proves a WPM broadband noise, nevertheless its amplitude is not a continuous normally distributed process, but is quantized over few discrete values (five in this case, at values evenly spaced 200 ps). Therefore, the Gaussian assumption holds only loosely.

In all the graphs shown, the agreement between the classical MTIE(τ) curves and the percentile $MTIE_{99\%}$ values, estimated under the assumption of Gaussian WPM noise, is quite good, considered especially the mix of different noise types revealed by measurements.

5. Conclusions

In this paper, the percentile MTIE was estimated, under the assumption of TE affected by Gaussian WPM noise, by deriving the probability distribution of the TE spanned range as a function of the noise standard deviation σ, thus allowing to interpret common Allan-variance factory specifications in terms of percentile MTIE as well. The measurement results shown confirmed that the formulas derived are very accurate, at least if the underlying noise can be well modelled as Gaussian WPM. Topics left for future research are thus the analysis of the percentile MTIE under other types of power-law noise (viz. flicker phase modulation and white, flicker and random-walk frequency modulation).

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