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Properties of the Traffic Output by a Leaky-Bucket Policer with Long-Range Dependent Input Traffic

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***Abstract* — Long-range dependence (LRD) is a largely verified property of Internet traffic, which severely affects queuing performance in network buffers. A common approach for guaranteeing performance requirements is to control the statistical profile of the input traffic by regulators based on the leaky bucket scheme. In this paper, we investigate by simulation how the $1/f^{\alpha}$ power-law spectrum of LRD traffic is altered when traffic is regulated by a leaky bucket policer. Analysis of the traffic spectral characteristics is carried out mainly by means of the Modified Allan Variance, a time-domain quantity with demonstrated superior accuracy in fractional-noise parameter estimation, recently introduced also for traffic analysis. This approach allows to get a finer insight into power-law spectral characteristics of policed traffic. We also investigate some other properties of the leaky bucket fed with LRD traffic, such as its dropping probability and its effect on queuing delay in a following FIFO scheduler.**

***Index Terms* — Communication system traffic, fractional noise, Internet, long-range dependence, queuing analysis, traffic control (communication).**

I. Introduction

Internet traffic exhibits temporal correlation properties, such as self-similarity and long memory (long-range dependence, LRD) on various time scales [1]—[3]. These properties emphasize long-range time-correlation between packet arrivals. Fractional noise and fractional Brownian motion models are often used to describe such behaviour of Internet traffic series, which include, but are not limited to, cumulative or incremental data count transmitted over time, inter-arrival time series of successive TCP connections or IP packets, etc.

In a self-similar random process, a dilated portion of a realization (sample path) has the same statistical characterization than the whole. "Dilating" is applied on both amplitude and time axes of the sample path, according to a scaling parameter H (Hurst parameter). On the other hand, LRD is a long-memory property, usually equated to an asymptotic power-law decrease of the power spectral density (PSD) $\sim f^{-\gamma}$ (for $f \rightarrow 0$) or, equivalently, of the autocovariance function. Under some hypotheses [2], the integral of a LRD process is self-similar with H related to γ (e.g., fractional Brownian motion, integral of fractional Gaussian noise).

It has been well recognized [4]—[7] that LRD in input traffic contributes to build up long queues in network buffers. In the case of fractional Gaussian traffic, for example, it has been found [4][5] that the queue tail has Weibull distribution, i.e. the buffer occupancy X exceeds a given threshold x with asymptotic probability $P\{X > x\} \sim exp(-\beta x^{1-\gamma})$, where β is a positive function of γ and of other network parameters.

The Weibull queue length distribution departs significantly from the plain exponential distribution associated to Poisson or short-range dependent input traffic. In particular, the closer γ is to 1, the slower the queue tail decreases. Consequently, the probability of suffering high waiting delay in network buffers increases dramatically. In conclusion, the network delay performance depends considerably on the actual value of the H and γ parameters, among others. Hence, a notable interest, dating back to 1990's, for appropriate analysis tools to characterize self-similar or LRD traffic and to estimate its parameters.

Guaranteeing performance requirements, e.g. delay limits, calls for a strict control of the statistical profile of the traffic offered to the network. A common approach is to control input traffic by regulators based on the *leaky bucket* scheme.

The leaky bucket regulator was proposed by J. Turner in [8]. This scheme is based on a non-negative counter, which is incremented whenever the controlled traffic flow offers a packet (or bit/byte) to the network and, if greater than zero, is decremented periodically with rate r. If the counter exceeds a given threshold b upon being incremented, the regulator drops the packet. Thus, the leaky bucket controls the *average rate* (enforced to r) and the *burstiness* (depending on b) of the through traffic. In other words, this scheme actually behaves as a *traffic policer*, because traffic is forwarded immediately, with no buffering, or simply dropped if parameters are exceeded.

Enforcing average rate and burstiness on input flows may allow to attain given performance targets in the network. The possibility of controlling input traffic by leaky bucket regulators has been widely treated in literature, since early works considering only Markovian or short-range dependent traffic [9]—[11]. Later on, several authors extended research to leaky-bucket regulation of LRD traffic [12]—[18]. Most studies have proven that it is difficult to reduce long-range dependence by use of leaky bucket regulators. Notwithstanding this general statement, precise conclusions drawn by some works are partially contradicting.

In paper [12], a study on measured and synthetic traffic traces concludes that it seems possible to reduce LRD of the regulated traffic, but only if the regulator drops traffic drastically. In paper [13], an analytical study is carried out on a type of LRD traffic consisting of connections with long-tailed duration, offered to the system as a Poisson arrival process. This

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paper concludes that LRD cannot be removed from the regulated traffic. However, a different result is presented in paper [14], which claims that it is possible to reduce traffic correlation (i.e., the Hurst parameter) by a shaping regulator. Finally, in papers [15][16], the output traffic of a single buffered server (rather than a regulator) is studied analytically with LRD input traffic. Both these studies conclude that the Hurst parameter of the output traffic is identical to that of the input traffic.

Such apparent contradictions mainly stem from the difficulty of studying analytically the traffic output by a traffic regulator, which is both non linear and provided with memory, fed with LRD input traffic. Simulation as well is made cumbersome by the asymptotical definition of LRD for $f \rightarrow 0$.

In this work, we investigated how LRD traffic is affected by a leaky bucket policer, as defined by scheme [8] recalled above. In particular, we simulated the behaviour of a leaky bucket fed with input traffic $x(t)$ [bit/s] having power-law one-sided PSD $S_x(f) = C/f^{\gamma}$ (for $0 < \gamma < 1$). Then, we studied the output traffic spectrum, observing how this is affected for various values of r and b. Analysis of the traffic spectral characteristics was carried out mainly by means of the Modified Allan Variance (MAVAR), a time-domain quantity originally conceived for frequency stability characterization, because of its demonstrated superior spectral sensitivity and accuracy in fractional-noise parameter estimation. We also investigated some other properties of the leaky bucket fed with LRD traffic, such as its dropping probability and its effect on queuing in a downstream First-In First-Out (FIFO) buffer.

The rest of the paper is organized as follows. In Sec. II, basic notions of self-similarity and long-range dependence are briefly summarized. In Sec. III, main properties of MAVAR for estimating LRD parameters are recalled, referring to a general power-law model of processed data. In Sec. IV, the input traffic model and the procedure for pseudo-random traffic generation are detailed. In Sec. V, simulations results are presented and commented.

II. Self-Similarity and Long-Range Dependence

A random process $X(t)$ (e.g., cumulative packet arrivals in time interval $[0, t]$), is said to be *self-similar*, with scaling parameter of self-similarity or Hurst parameter $H>0$, $H\in\Re$, if

$$X(t) =_d a^{-H} X(at) \tag{1}$$

for any $a>0$, where $=_d$ denotes equality for all finite-dimensional distributions [1][2]. In other terms, the statistical description of $X(t)$ does not change by *scaling* simultaneously its amplitude by a^{-H} and the time axis by a. Most self-similar processes are not stationary, since the moments of $X(t)$, provided they exist, behave as power laws of time [1].

In practice, the class of self-similar (H-SS) processes is usually restricted to that of *self-similar processes with stationary increments* (or H-SSSI processes), which are "integral" of some stationary process. For example, consider the δ-increment process of $X(t)$, defined as $Y_\delta(t) = X(t)-X(t-\delta)$ (e.g., packet arrivals in the last δ time units). For a H-SSSI process $X(t)$, $Y_\delta(t)$ is stationary and $0 < H < 1$ [2].

Long-range dependence (LRD) of a process is defined by an asymptotic power-law decrease of its autocovariance or equivalently PSD functions [1][2]. Let $Y(t)$ be a second-order stationary stochastic process. The process $Y(t)$ exhibits LRD if its autocovariance function follows asymptotically

$$R_Y(\delta) \sim c_1|\tau|^{\gamma-1} \quad \text{for } \tau \rightarrow +\infty\,,\ 0<\gamma<1 \tag{2}$$

or, equivalently, its two-sided power spectral density (PSD) follows asymptotically

$$S_Y(f) \sim c_2|f|^{-\gamma} \quad \text{for } f \rightarrow 0\,,\ 0<\gamma<1 \qquad (3).$$

In general, a random process with non-integer power-law PSD is also known as fractional (not necessarily Gaussian) noise. It can be proven [2] that H-SSSI processes $X(t)$ with $1/2 < H < 1$ have long-range dependent increments $Y(t)$, with

$$\gamma = 2H - 1 \qquad (4).$$

Strictly speaking, the Hurst parameter characterizes self-similar processes, but it is frequently used to label also the LRD increments of H-SSSI processes. In this paper, we follow this common custom with no ambiguity. Hence, the expression "Hurst parameter of a LRD process" (characterized by parameter γ) denotes actually, by extension, the Hurst parameter $H = (\gamma+1)/2$ of its integral H-SSSI parent process.

III. Estimating Parameters H and γ of LRD Data Using the Modified Allan Variance

Estimating statistical parameters that characterize self-similar and LRD random processes is an issue well studied in literature [1][2][19][20]. In this work, we used the Modified Allan Variance (MAVAR), recently introduced as traffic analysis tool for very accurate estimation of parameters H and γ of given self-similar and LRD traffic series [21][22].

A. The Modified Allan Variance

MAVAR is a well-known time-domain quantity, originally conceived in 1981 for frequency stability characterization of precision oscillators [23]—[27] by modifying the definition of the Allan Variance (AVAR) recommended by IEEE in 1971 [28]. MAVAR was designed with the goal of discriminating noise types with power-law spectrum (i.e., in broad terms, fractional noise) recognized very commonly in frequency sources. MAVAR has been demonstrated to feature superior spectral sensitivity and accuracy in fractional-noise parameter estimation, coupled with excellent robustness against nonstationarities in data analyzed [22] (e.g., drift and steps). MAVAR was also successfully applied to real network traffic analysis, allowing to identify fractional noise in experimental results [22][29]. This section briefly recalls MAVAR properties most relevant to our aim. For all details, the interested reader is referred to the bibliography.

Given a finite set of N samples $\{x_k\}$ of a signal $x(t)$, evenly spaced by sampling period τ_0, MAVAR can be estimated using the ITU-T standard estimator [23]

$$\mathrm{Mod}\,\sigma_y^2(\tau) = \frac{\displaystyle\sum_{j=1}^{N-3n+1}\left[\sum_{i=j}^{n+j-1}\left(x_{i+2n} - 2x_{i+n} + x_i\right)\right]^2}{2n^4\tau_0^2\left(N-3n+1\right)} \tag{5}$$

where the observation interval is $\tau = n\tau_0$ and $n = 1, 2, \ldots, \lfloor N/3 \rfloor$.

The MAVAR is a kind of variance of the second difference of input data. In very brief, it differs from the unmodified Allan variance in the additional internal average over n adjacent samples: for n=1 ($\tau=\tau_0$), the two variances coincide. A recursive algorithm for fast computation of this estimator exists [23], which cuts down the number of operations needed for all values of n to $\sim N^2$ instead of $\sim N^3$.

It should be noted that the point estimate (5), computed by averaging N-3n+1 terms, is a random variable itself. Exact computation of confidence intervals is not immediate and, annoyingly enough, depends on the spectrum of the underlying noise [30]—[34]. However, in general, along a plot of MAVAR(τ), confidence intervals are negligible for short τ and widen moving to longer τ, where fewer terms are averaged. In our results, therefore, we excluded MAVAR values computed for largest n, where uncertainty is not negligible.

B. Power-Law Random Processes

It is convenient to extend the LRD power-law model of PSD (3). As customary in characterization of phase and frequency noise of precision oscillators [35], we deal with random processes $x(t)$ whose one-sided PSD is modelled as

$$S_x(f) = \begin{cases} \sum_{i=1}^{P} h_{\alpha_i} / f^{\alpha_i} & 0 < f \le f_\mathrm{h} \\ 0 & f > f_\mathrm{h} \end{cases} \tag{6}$$

where P is the number of noise types considered, α_i and h_{α_i} are model parameters (α_i, $h_{\alpha_i} \in \Re$) and f_h is the upper cut-off frequency. Such random processes are commonly referred to as *power-law* or *fractional noise* (not necessarily Gaussian).

Power-law noise with $0 \le \alpha_i \le 4$ was revealed in practical measurements of various physical phenomena, such as phase noise of precision oscillators [23][28][35] and Internet traffic [1][2][29], whereas P should be not greater than few units for the model being useful. If the process $x(t)$ is LRD with PSD (3), then this model still applies, for P=1 and $0 < \alpha_i < 1$ (at least asymptotically). Although values $\alpha_i \ge 1$ yield model pathologies, such as infinite variance and even nonstationarity [36], this model is common, considering also that real-world measurements have finite duration and bandwidth.

Under this general hypothesis of power-law PSD, by letting P=1, $\alpha=\alpha_i$ and in the whole range of MAVAR convergence $0 \le \alpha < 5$, MAVAR is found to follow a simple power law (ideally asymptotically for $n\to\infty$, $n\tau_0=\tau$, but in practice for n>4), i.e.

$$\mathrm{Mod}\,\sigma_y^2(\tau) \sim A_\mu \tau^\mu \;, \quad \mu = -3 + \alpha \tag{7}$$

If P>1, it is immediate to generalize (7) to summation of powers $\sum_i A_{\mu_i} \tau^{\mu_i}$. This is a fundamental result. If $x(t)$ obeys (6), a log-log plot of Mod $\sigma_y^2(\tau)$ looks ideally as a piecewise function made of P straight segments, assuming sufficient separation between components, whose slopes μ_i can be estimated to yield exponents $\alpha_i = 3+\mu_i$ of the fractional noise terms that are dominant in different ranges of τ.

If we consider a LRD process with PSD (3), characterized by Hurst parameter $1/2 < H < 1$, from (4) and (7) we obtain

$$\begin{aligned} H &= \mu/2 + 2 \\ \gamma &= \alpha = \mu + 3 \end{aligned} \tag{8}$$

In paper [22], these estimates of H and α were demonstrated to be very accurate and robust against nonstationarities in the processed data (drifts, periodic trends and steps).

Finally, let us notice that this procedure is analogous to that of the wavelet second-order log-scale diagram technique [1][2][20], which analyzes data over a range of scales, by observing the power-law behaviour (i.e., estimating the slopes) of the wavelet detail variances across octaves.

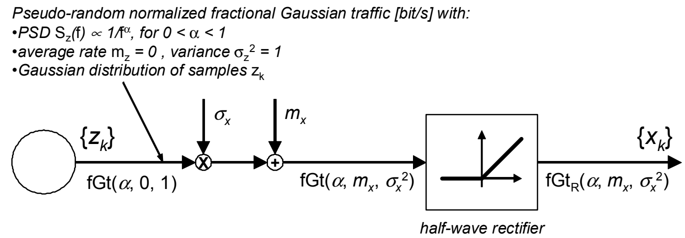

Fig. 1: Generation of rectified fractional Gaussian traffic.

IV. Model and Synthesis of Input Traffic

Several methods to generate pseudo-random self-similar and LRD data sequences are available in literature. In this work, we followed the method proposed by Paxson [37] for fast generation of *fractional Gaussian traffic* (fGt). This type of traffic is remarkable, since when it is fed into a FIFO buffer the queue tail distribution can be derived analytically [4][5].

The Paxson's procedure generates LRD pseudo-random data series $\{x_k\}$ of length N, with power-law one-sided PSD $S_x(f) = h/f^\alpha$, for assigned values of α ($0 < \alpha < 1$) or $H = (1+\alpha)/2$, mean m_x and variance σ_x^2. The sequence $\{x_k\}$ represents the incremental data count [bit/s] input at each time unit into the leaky bucket under study. The procedure is outlined in Fig. 1.

First, a pseudo-random sequence $\{z_k\}$ denoted fGt(α, 0, 1) is generated, with PSD $S_z(f) \propto 1/f^\alpha$, normally-distributed samples, null mean m_z=0 and variance σ_z^2=1. Then, in order to adjust its mean and variance, fGt(α, 0, 1) is multiplied by σ_x and added to m_x, obtaining the sequence fGt(α, m_x, σ_x^2). In order to avoid negative samples (data rate cannot be negative), fGt(α, m_x, σ_x^2) is then filtered by a half-wave rectifier, which leaves untouched the positive samples, but outputs a null sample when the input is negative. Finally, we obtain the "rectified" fractional Gaussian traffic sequence fGt$_R$(α, m_x, σ_x^2), i.e. $\{x_k\}$.

Obviously, both mean and variance of fGt$_R$ are slightly smaller than the target values m_x and σ_x^2 of fGt(α, m_x, σ_x^2). Moreover, the half-wave rectifier is a non linear device, which unfortunately distorts the power spectrum of the fGt sequence. The analysis of the rectified process spectrum is difficult [38], but it can be proven that α is somehow altered.

Anyway, the distortion of α, m_x and σ_x^2 is negligible, if the half-wave rectifier clips negative samples only rarely. Since the probability of having a negative sample in the fGt sequence is equal to

$$\frac{1}{2}\left[1-erf\left(\frac{m_x}{\sqrt{2}\sigma_x}\right)\right] \tag{9}$$

parameters m_x and σ_x^2 can be set to make negative samples to occur very seldom in fGt(α, m_x, σ_x^2). Therefore, it is not actually possible to set the m_x and σ_x^2 parameters completely at will: the standard deviation σ_x must be small enough, with respect to m_x, to limit spectral distortion by the rectifier.

In simulations presented in this paper, however, we set carefully the mean and variance of the fGt series, to ensure that the occurrence of negative samples is very rare. Moreover, the m_x, σ_x^2 and α parameters of the rectified series fGt$_R$ were also measured *a posteriori*: in all cases, they resulted practically equal to the corresponding values of the original fGt series.

It is worth noting that also other methods are available in literature to generate pseudo-random fractional traffic. For example, paper [39] presents a method for synthesizing log-normal multi-fractal traffic sequences. Log-normal processes have the desirable property of being non-negative by construction, which relieves the need of rectifying the sequence as needed for Gaussian series. Moreover, the multi-fractal model allows also to consider traffic with more complex behaviour than a simple power law. In this work, nevertheless, we decided to adopt the fractional Gaussian traffic model for the sake of simplicity and because, as pointed out before, for this type of traffic the tail distribution of a FIFO queue was derived analytically [4][5]. Further simulations, with multi-fractal log-normal traffic, are left for future work.

V. Simulation Results

According to the Paxson's procedure outlined in the previous section, we generated sequences of fractional Gaussian traffic $\{x_k\}$ made of $N = 2^{23} = 8\,388\,608$ samples, representing the incremental data count [bit/s] input at each time unit into the leaky bucket under study. We set the time unit $\tau_0 = 1$ ms, the mean $m_x = 2279$ bit per time unit (i.e., 2.279 Mbit/s) and the deviation $\sigma_x = 773.9$ bit per time unit (i.e., 773.9 kbit/s), as in [5]. We varied α in range $0 < \alpha < 1$. For example, Fig. 2 plots a segment of fGt$_R$ series $\{x_k\}$ generated with $\alpha = 0.50$.

The input traffic $x(t)$ was fed into a leaky bucket policer, as defined by scheme [8] recalled in Sec. I. Then, we characterized the output traffic, observing how it is affected for various values of the policer rate r [bit/s] and threshold b [bit], both in the time domain, by means of MAVAR, and in the frequency domain, by classic FFT-based power spectrum estimation (periodogram over 1024 points, having divided the sequence in 8192 segments with Welch data windowing [40]).

Figs. 3 and 4 show the PSD and MAVAR, respectively, computed on the traffic sequence at the output of the policer, with threshold $b = 14202$ bit and for various values of the ratio r/m_x of the policer rate to the input traffic mean rate, fed with fGt$_R$ input traffic with $\alpha = \alpha_{IN} = 0.50$.

Curves for $r/m_x = \infty$ were computed directly on the input sequence $x(t)$, which in this case transits through the policer unaffected, as obvious. The value $r/m_x = 1.2$ may represent a normal operation condition of the leaky bucket, when the policer rate is greater than the source mean rate and the policer drops traffic only rarely. In this case, the customer is complying with the traffic conditioning agreement and the policer does not clip traffic significantly: both PSD and MAVAR of the output traffic nearly coincide with those of input traffic. Decreasing the ratio r/m_x, we notice that the spectral characteristics of the through traffic begin to be affected significantly.

When $r/m_x \geq 1$ or so, the policer still works in a quasi-linear way: the output PSD and MAVAR still follow approximately a simple power law (linear trend in the log-log plot), i.e. the output traffic still obeys the model (6) with $P = 1$, $S_x(f) \cong h/f^{\alpha}$, although with different α. In other words, the policer is altering LRD parameters α and H of the through traffic, but it is not distorting significantly the spectral power-law nature of the fractional traffic.

When $r/m_x << 1$, the policer drops a significant part of traffic and the effect of its nonlinear behaviour becomes apparent: the output PSD and MAVAR depart from a simple power law. The minimum value of the ratio we considered is $r/m_x = 0.02$: in this extreme case, the policer drops ~97% of the traffic. In practice, such a situation may occur when the customer exceeds dramatically the limits of the traffic contract and the policer drops almost the entire traffic offered. In this case, the output traffic spectrum does not obey anymore the simple power law (6) with $P = 1$: the traffic spectrum now appears approximable by a two-terms power-law model ($P = 2$), i.e. $S_x(f) \cong h/f^{\alpha_1} + h/f^{\alpha_2}$. It is worth noticing that this is better evident in the MAVAR curve, rather than in the PSD, since low-frequency terms are "expanded" (this is the reason why time-domain quantities are better suited than frequency-domain measures to analyze fractional noise with $\alpha > 0$).

A closer look on Fig. 4 calls for a more thorough investigation on how the α parameter of LRD traffic is altered through the leaky bucket. MAVAR is particularly suitable for such a study, because of its superior accuracy in fractional-noise parameter estimation, as widely demonstrated in cited literature.

By inspection of Fig. 4, first we notice that for $r/m_x \geq 0.4$ the MAVAR trend is nearly linear and thus the output traffic spectrum can be somehow approximated by a simple power law $S_x(f) \cong h/f^{\alpha}$ ($P = 1$). For $r/m_x < 0.4$, the traffic spectrum is heavily distorted by the policer and now includes two clear power-law terms, i.e. $S_x(f) \cong h/f^{\alpha_1} + h/f^{\alpha_2}$ ($P = 2$). Actual values of α_1 and α_2, estimated by linear regression separately in intervals 10^{-3} s $< \tau < 3\cdot10^{-1}$ s and 10^{0} s $< \tau < 3\cdot10^{2}$ s, respectively, are reported in Table 1.

To summarize, the lower is the ratio r/m_x (i.e., the more traffic is clipped), the more the α parameter is diminished by the leaky bucket action. However, when the ratio r/m_x is very low, the strong clipping action of the policer distorts the spectrum so much, to produce a new, slower power-law term, with $\alpha>1$. For $r/m_x \geq 0.4$, the estimated values of α_1 and α_2 are very close, being MAVAR(τ) almost linear.

As a final, perhaps unnecessary remark, we mention that altering the parameter α of a fractional random process with PSD $S_x(f) = h/f^{\alpha}$ can be seen as filtering the signal through an integrator or differentiator of *fractional order* [41].

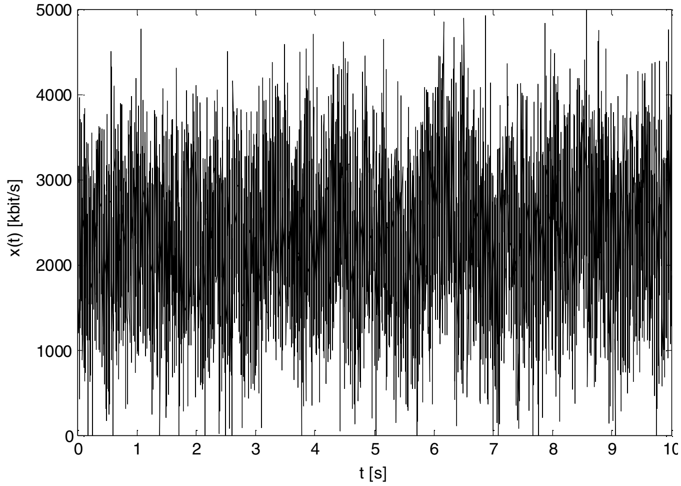

Fig. 2: Segment of fGt_R sequence $\{x_k\}$ generated by the method in Fig. 1 (α=0.50, m_x=2.279 kbit/ms, σ_x=773.9 bit/ms, τ_0=1 ms).

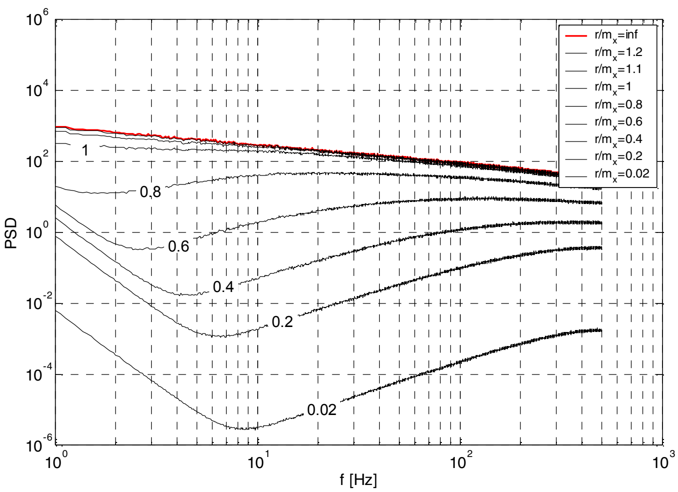

Fig. 3: PSD of the traffic output by a leaky bucket policer (b=14202 bit, r) with fGt_R input traffic (α_{IN}=0.50, m_x=2.279 kbit/ms, σ_x=773.9 bit/ms, τ_0=1 ms).

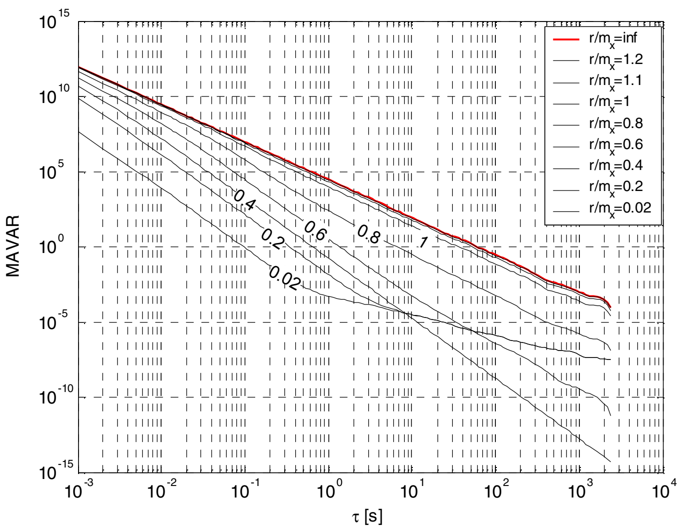

Fig. 4: MAVAR of the traffic output by a leaky bucket policer (b=14202 bit, r) with fGt_R input traffic (α_{IN}=0.50, m_x=2.279 kbit/ms, σ_x=773.9 bit/ms, τ_0=1 ms).

Table 1:
Values of α_1 and α_2 estimated from MAVAR results in Fig. 4 (α_{IN}=0.50).

r/m_x	α_1 10^{-3} s < τ < $3\cdot10^{-1}$ s	α_2 10^{0} s < τ < $3\cdot10^{2}$ s
∞	0.50	0.489
1.2	0.486	0.488
1.1	0.438	0.482
1.0	0.325	0.449
0.8	-0.116	0.201
0.6	-0.612	-0.345
0.4	-0.868	-0.998
0.2	-0.957	1.255
0.02	-0.921	1.655

The case $r/m_x \ll 1$, with high clipping rate in the policer and heavy traffic spectrum distortion, corresponds to a scenario that, in practice, should occur only exceptionally. Therefore, we investigated more thoroughly the behaviour of the leaky bucket in case $r/m_x > 1$, in which the output traffic spectrum can be still well approximated by a simple power law, attempting to study how the α parameter of LRD traffic is altered by the policer.

Simulation results shown in Figs. 5 and 6 were obtained by feeding the leaky bucket with fGt_R traffic with $\alpha = \alpha_{IN} = 0.90$. Then, we varied both parameters r and b of the policer in a wide interval. The α parameter of the output traffic was estimated by linear regression on MAVAR curves (we excluded safely the last 2 decades because of lower confidence).

Fig. 5 plots the *loci* of the (r, b) pairs, for which the same value α_{OUT} of the α parameter was estimated on the output traffic, having normalized r to the input traffic average m_x and b to the input traffic deviation σ_x. From this graph, we can observe that the dependence of α_{OUT} on the leaky bucket parameters is not trivial. Apparently, for relatively large values of r/m_x and b/σ_x, we have $\alpha_{OUT} \cong \alpha_{IN}$ and in general α_{OUT} decreases as r/m_x and b/σ_x get smaller. However, it is also possible to observe the non-monotonic behaviour of α. For example, we measured α_{OUT} = 0.86 with r/m_x = 1.23 for three different values of b/σ_x (points A1, A2 and A3).

Fig. 6 plots, for the same model settings as in Fig. 5, the fraction of traffic dropped by the regulator. The dropping probability follows a much more intuitive trend, that is, it always diminishes when either r/m_x or b/σ_x grows and vice-versa. These results confirm that the dependence of α of the output traffic on leaky bucket settings has significantly higher complexity than more traditional performance measures, such as the dropping probability.

As recalled in Sec. I, the α parameter of input traffic has great importance for the provisioning of network resources. Therefore, we simulated a scenario where the policer output traffic is fed into a FIFO scheduler. The $x(t)$ traffic at the input of the leaky bucket has the same average rate m_x and deviation σ_x set in previous experiments. The policer rate and threshold are set r = 3 Mbit/s (i.e., r/m_x = 1.31) and b = 14202 bit (i.e., b/σ_x = 18.3 ms), respectively. With these settings, the policer affects a negligibly (cf. Fig. 4 and Table 1). The FIFO scheduler has an output line with capacity C = 2.532 Mbit/s.

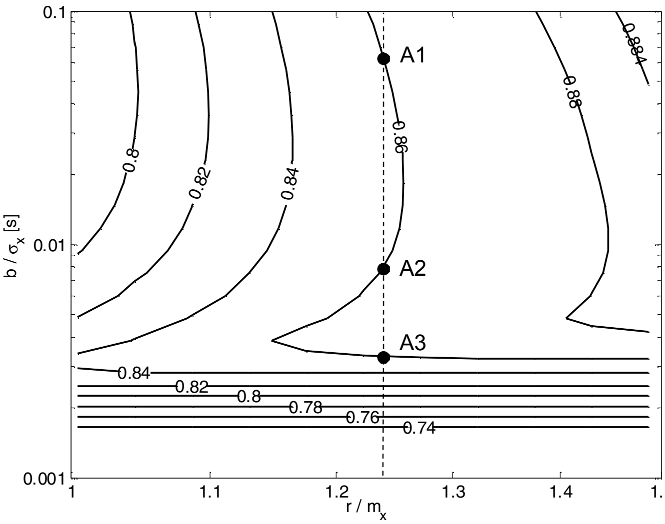

Fig. 5: *Loci* of the (r/m_x, b/σ_x) pairs for which the same α_{OUT} was estimated on the traffic output by a leaky bucket policer fed with fGt_R (α_{IN}=0.90).

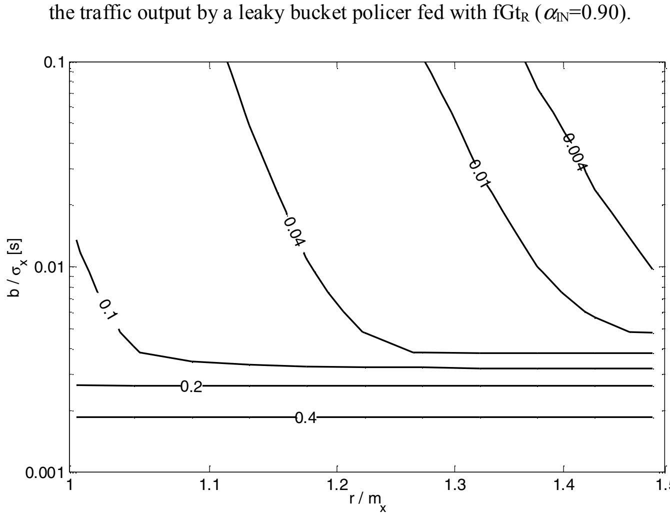

Fig. 6: *Loci* of the (r/m_x, b/σ_x) pairs for which the same dropping probability was measured in a leaky bucket policer fed with fGt_R (α_{IN}=0.90).

Then, Fig. 7 plots the probability $P(d>D)$, estimated by simulation, that the traffic experiences a delay d greater than D in the FIFO queue, for four different values of the α parameter of the policer input traffic, viz. $\alpha = 0.0$, 0.5 and 0.8.

Let us assume that the network operator and the customer stipulated a traffic conditioning agreement with $r = 3$ Mbit/s and $b = 14202$ bit. Moreover, the service level agreement specifies that the probability that the delay d in the scheduler exceeds $D = 20$ ms is $P(d>20 \text{ ms}) \leq 0.05$. Finally, let us assume that the customer supplies fGt_R traffic $x(t)$ with m_x and σ_x as in previous simulations, with $\alpha = 0.5$. By inspection of Fig. 7, we conclude that the service level agreement is fulfilled, because $P(d>20 \text{ ms}) < 0.05$. However, if the customer supplies $x(t)$ with same m_x and σ_x, but with $\alpha = 0.8$, we observe again from Fig. 7 that the service level agreement is now violated, as the probability of exceeding delay $D = 20$ ms results $P(d>20 \text{ ms}) \cong 0.45$. In this case, the policer is unable to alter the α parameter of traffic and the result is a disruption of the required quality of service.

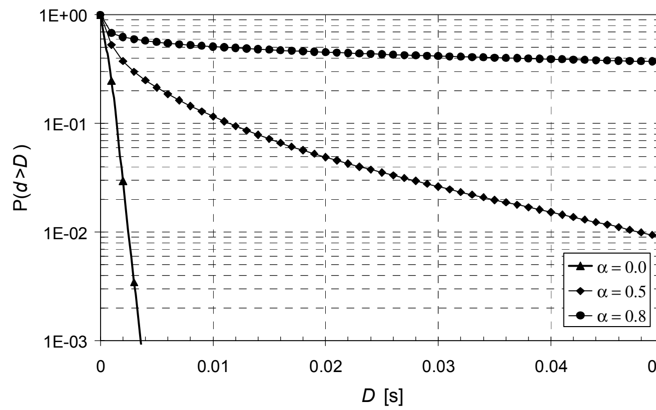

Fig. 7: Probability of exceeding delay D in a FIFO queue at the output of the policer (m_x=2.279 kbit/ms, σ_x=773.9 bit/ms, τ_0=1 ms).

VI. Conclusions

In this paper, we investigated by simulation how the $1/f^{\alpha}$ power-law spectrum of LRD traffic is altered when traffic is regulated by a leaky bucket policer. Analysis of the traffic spectral characteristics was carried out mainly in the time domain by means of the Modified Allan Variance, because of its demonstrated superior accuracy in fractional-noise parameter estimation. This approach allowed to get a finer insight into power-law spectral characteristics of regulated traffic.

In our simulations, we found that the policer may alter significantly the spectral characteristics of through traffic, depending in particular on ratio r/m_x (policer rate to input traffic mean rate). Our findings, at least to the limited extent of the hypotheses made, may be summarized as follows:

- when $r/m_x \geq 1.2$, traffic transits practically unchanged;
- when $1 \leq r/m_x < 1.2$ or so, the output traffic spectrum is still well approximated by a simple power law h/f^{α}, i.e. the leaky bucket does not distort significantly the spectral $1/f^{\alpha}$ nature of the fractional traffic; nevertheless, not to mention the impact on m_x and σ_x^2, traffic LRD is reduced, as the α parameter appears diminished by policer clipping;
- when $r/m_x < 0.4$, the policer drops a significant part of traffic and the effect of its nonlinear behaviour becomes apparent: the output spectrum, heavily distorted by the policer, departs from a simple power law and includes two terms, i.e. $S_x(f) \cong h/f^{\alpha_1} + h/f^{\alpha_2}$, dominant in different intervals of frequency f or observation interval τ; while α_1 is smaller than the input α, α_2 is considerably higher (even >1);
- to summarize, the lower is r/m_x, the more the α parameter is diminished by the policer; however, when r/m_x is very low, policer clipping heavily distorts the traffic spectrum, which departs from the simple LRD model, and the effect on α may be hard to predict.

To conclude, this study confirms that controlling the α parameter of source traffic by use of a leaky bucket policer is difficult, if not impossible, aiming at guaranteeing the delay performance specified in service level agreements. In fact, if the customer increases the mean rate of its offered traffic, the leaky bucket is able to clip it effectively, thus avoiding network congestion. On the other hand, if the customer increases the α parameter of its offered traffic, the policer may very well let it pass unaltered and, in turn, large queues and delay may build up in downstream queues.

Our research activity is now focused on a more complete characterization of the output of this and other types of traffic regulators, aiming at identifying schemes capable of acting more effectively on the α parameter of regulated traffic.

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