Improved Estimation of the Hurst Parameter of Long-Range Dependent Traffic Using the Modified Hadamard Variance

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***Abstract* — Internet traffic exhibits self-similarity and long-range dependence (LRD) on various time scales. In this paper, we propose to use the Modified Hadamard Variance (MHVAR), a time-domain measure for high-resolution spectral analysis, to estimate the Hurst parameter *H* of LRD traffic data series or, more generally, the exponent α of traffic series with $1/f^{\alpha}$ power-law spectrum. MHVAR generalizes the principle of the Modified Allan Variance (MAVAR), a well-known tool widely used since 1981 for frequency stability characterization, to higher-order differences of input data; in our knowledge, it has been mentioned in literature only few times and with little detail so far.**

The behaviour of MHVAR with power-law random processes and some common deterministic signals (viz. drifts, sine waves, steps) is studied. The MHVAR performance in estimating *H* is evaluated by analysis and simulation, comparing it to the wavelet Logscale Diagram (LD) and to MAVAR. Extensive simulations show that MHVAR has highest accuracy and confidence in fractional-noise parameter estimation, even slightly better than MAVAR. Moreover, MHVAR features a number of other advantages, which make it useful to complement other established techniques such as MAVAR and LD. Finally, MHVAR and LD are also applied to a real IP traffic trace.

***Index Terms* — Fractional Brownian motion, fractional noise, Internet, long-range dependence, random walk, self-similarity, traffic control (communication), traffic model.**

I. Introduction

Internet traffic exhibits intriguing temporal correlation properties, such as self-similarity and long memory (long-range dependence) on various time scales [1]—[3]. Contrary to the classical Poisson-model assumption, these properties emphasize long-range time-correlation between packet arrivals. Fractional noise models are used to describe the behaviour of such traffic series, which include, but are not limited to, cumulative or incremental data count transmitted over time, inter-arrival time series of successive TCP connections or IP packets, etc.

In a self-similar random process, a dilated portion of a realization (sample path) has the same statistical characterization than the whole. "Dilating" is applied on both amplitude and time axes of the sample path, according to a scaling parameter called Hurst parameter. On the other hand, long-range dependence (LRD) is a long-memory property observed on large time scales: LRD is usually equated with an asymptotic power-law decrease of the autocovariance function and of the power spectral density (PSD). Under some hypotheses, the integral of a LRD process is self-similar (e.g., fractional Brownian motion, integral of fractional Gaussian noise).

Estimating statistical parameters that characterize self-similar and LRD random processes is a well studied issue, aiming at best modelling traffic for example to the purpose of network simulation. Several algorithms exist, in particular, to estimate the Hurst parameter *H* and the spectrum frequency exponent γ of given self-similar and LRD traffic series [1]-[5].

In previous papers [6][7], the Modified Allan Variance (MAVAR) was introduced as traffic analysis tool for accurate estimation of *H* and γ. MAVAR is a well-known time-domain quantity, originally conceived in 1981 for frequency stability characterization of precision oscillators [8]—[11] by modifying the definition of the Allan Variance (AVAR) recommended by IEEE in 1971 [12]. MAVAR was designed with the goal of discriminating noise types with power-law spectrum (i.e., in broad terms, fractional noise) recognized very commonly in frequency sources. MAVAR has been demonstrated to feature superior spectral sensitivity and accuracy in fractional-noise parameter estimation, coupled with robustness against nonstationarity in data analyzed [6][7]. MAVAR was successfully applied to real network traffic analysis, allowing to identify fractional noise in experimental results [6][7][13].

In this work, we extended the scope of research [6], investigating the properties of other time-domain variances designed after AVAR, with the aim at further improving the accuracy of estimation of *H* and γ. Then, a Modified Hadamard Variance (MHVAR) is studied by analysis and simulation. In our knowledge, this particular variance has been mentioned in literature only few times and with little detail so far.

Extensive simulations show that MHVAR exhibits the highest accuracy and confidence in fractional-noise parameter estimation, even slightly better than MAVAR. The MHVAR method has been applied to pseudo-random LRD data series and evaluated by comparison to the well-established logscale diagram (LD) technique based on wavelet analysis [2][4] and to the MAVAR method previously proposed in [6][7]. Moreover, the behaviour of MHVAR with some deterministic signals that yield nonstationarity in data under analysis is addressed. Finally, MHVAR and LD are also evaluated on a real IP traffic trace, providing a sound example of application to experimental traffic characterization.

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II. Self-Similarity and Long-Range Dependence

A random process $X(t)$ (say, cumulative packet arrivals in the time interval $[0, t]$) is said to be *self-similar*, with scaling parameter of self-similarity or Hurst parameter $H>0$, if

$$X(t) =^{d} a^{-H} X(at) \tag{1}$$

for all $a>0$, where $=^{d}$ denotes equality for all finite-dimensional distributions [1][2]. In other terms, the statistical description of $X(t)$ does not change by *scaling* simultaneously its amplitude by a^{-H} and time by a. Self-similar processes are not stationary by definition, since the moments of $X(t)$, provided they exist, behave as power laws of time.

In practical applications, the class of self-similar (H-SS) processes is usually restricted to that of *self-similar processes with stationary increments* (or H-SSSI processes), which are "integral" of some stationary process. For example, consider the δ-increment process of $X(t)$, defined as $Y_\delta(t) = X(t)-X(t-\delta)$ (say, packet arrivals in the last δ time units). For a H-SSSI process $X(t)$, $Y_\delta(t)$ is stationary and $0 < H < 1$ [2].

Long-range dependence (LRD) of a process is defined by an asymptotic power-law decrease of its autocovariance or equivalently PSD functions [1][2]. Let $Y(t)$ be a second-order stationary stochastic process. The process $Y(t)$ exhibits LRD if its autocovariance function follows asymptotically

$$R_Y(\delta) \sim c_1 |\delta|^{\gamma-1} \quad \text{for } \delta \to +\infty,\ 0<\gamma<1 \tag{2}$$

or, equivalently, its power spectral density (PSD) follows asymptotically

$$S_Y(f) \sim c_2 |f|^{-\gamma} \quad \text{for } f \to 0,\ 0<\gamma<1 \tag{3}$$

A random process with non-integer power-law PSD is also known as fractional noise. It can be proven [2] that H-SSSI processes $X(t)$ with $1/2 < H < 1$ have long-range dependent increments $Y(t)$, with

$$\gamma = 2H - 1 \tag{4}$$

Strictly speaking, the Hurst parameter characterizes self-similar processes, but it is frequently used to label also the long-range dependent increment processes of H-SSSI processes. Hence, the expression "Hurst parameter of a LRD process" (characterized by the parameter γ) denotes actually, by extension, the Hurst parameter $H = (\gamma+1)/2$ of its integral H-SSSI parent process.

III. Beyond The Modified Allan Variance

Although MAVAR proved very sensitive and robust in fractional-noise parameter estimation [6][7], yet requiring lightweight computational effort, we extended our scope of research studying other variances with even higher spectral resolution, aiming at further improving the accuracy of estimation of H and γ, but still with reasonable complexity. In this section, the most interesting variances considered in our study, by analysis and simulation, are overviewed. For best understanding of this and next sections, we assume that the reader is familiar with fundamentals of time-domain variances for phase and frequency stability characterization. For a basic survey on this subject, see [8][14]. More in detail, an extensive list of references is provided by [15].

Total Variance (TOTVAR) and *Modified Total Variance* are improvements of conventional estimators of the Allan Variance $\sigma^2_y(\tau)$ and Modified Allan Variance Mod $\sigma^2_y(\tau)$ [16]—[19]. Total estimators improve the confidence of the variance estimate for largest observation intervals τ, where few samples are averaged, by periodically extending the input data sequence beyond its finite measurement interval. Unfortunately, total estimators suffer bias, depending on τ and the type of underlying noise, which affects the curve slope in log-log diagrams. In practice, taking this bias into account makes cumbersome to estimate H and γ from total variance slope. Hence, total estimators are not suitable to our aim.

The *Hadamard Variance* (HVAR) was proposed by Baugh [20] in 1971, purposely for high-resolution spectral analysis. Generally based on a linear combination of M+1 consecutive samples, HVAR may attain highest spectral resolution, adjusting appropriately the dead time between measurements and the weighting coefficients of the M+1 samples [14]. In particular, the most useful definition of HVAR is based on weighting the M+1 samples with binomial coefficients (BC). This way, better spectral selectivity than AVAR is achieved [20][21]. The (M+1)-samples BC-weighted HVAR is a variance of the M^{th} difference of input data, whereas AVAR is a 2^{nd}-difference variance (i.e., based on 3 samples). The structure function theory, developed by Lindsey and Chie [22], gives a unifying view of such time-domain variances evaluated on the M^{th} difference of the data sequence analyzed.

A *Total Hadamard Variance* (TOTHVAR) has been defined as well, similarly to other total variances [21][23]. Also in this case, the total estimator improves confidence for largest observation intervals τ, but suffers bias that makes cumbersome to estimate H and γ in practice. Hence, also TOTHVAR is not suitable to our purpose.

In spite of its highest spectral resolution, HVAR is not able to discriminate effectively white from flicker ($1/f$) noise, similarly to AVAR. This makes plain HVAR not suitable to our purpose. Therefore, a *Modified Hadamard Variance* (MHVAR) is proposed and studied in this paper. MHVAR is derived by modifying the definition of BC-weighted HVAR analogously to MAVAR. In our knowledge, such a modified HVAR has been mentioned in literature only few times and expounded with little detail (a.k.a. "pulsar variance") [24][25].

IV. The Modified Hadamard Variance

This section summarizes some of most relevant MHVAR properties. Moreover, its behaviour with power-law random processes and some common deterministic signals is studied.

A. Definition and Estimator in the Time Domain

MHVAR generalizes the principle of MAVAR to higher-order differences of input data. Given an infinite sequence $\{x_k\}$ of samples of a signal $x(t)$, evenly spaced in time with sampling period τ_0, the MHVAR of order M (MHVAR-M) is defined as

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$$\mathrm{Mod}\,\sigma^2_{\mathrm{H},M}(\tau)=\frac{1}{M!\,n^2\tau_0^2}\left\langle\left[\frac{1}{n}\sum_{j=1}^{n}\sum_{k=0}^{M}\binom{M}{k}(-1)^k x_{j+kn}\right]^2\right\rangle \quad (5)$$

where the operator <·> denotes infinite-time averaging and $\tau=n\tau_0$ is the observation interval.

In brief, unmodified HVAR of order M is a kind of variance of the M^{th} difference of input data (but note the division by τ^2 instead of τ^{2M}). MHVAR differs from HVAR in the additional internal average over n adjacent samples: for n=1 ($\tau=\tau_0$), the two variances coincide. Moreover, let us note that, for M=2, MHVAR coincides with MAVAR. Most formulas in this section are generalizations of MAVAR formulas [7][8] with M as parameter. In most cases, HVAR and MHVAR of order M=3 have been considered in literature [14][20][21][23]—[25].

In practice, given a finite set of N samples $\{x_k\}$, again spaced by sampling period τ_0, MHVAR can be estimated as

$$\mathrm{Mod}\,\sigma^2_{\mathrm{H},M}(\tau)=\frac{\displaystyle\sum_{i=1}^{N-(M+1)n+1}\left[\sum_{j=i}^{i+n-1}\sum_{k=0}^{M}\binom{M}{k}(-1)^k x_{j+kn}\right]^2}{M!\,n^4\tau_0^2\,[N-(M+1)n+1]} \quad (6)$$

with n = 1, 2, ..., $N/(M+1)$.

It should be noted that the point estimate (6), computed by averaging N-(M+1)n+1 terms, is a random variable itself. Exact computation of confidence intervals is not immediate and, annoyingly enough, depends on the spectrum of the underlying noise [21][24][26]—[28]. However, in general, along a plot of Mod $\sigma^2_{H,\mathrm{M}}(\tau)$, confidence intervals are negligible at short τ and widen moving to longer τ, where fewer terms are averaged. In practice, being N usually in the order of 10^4 and above, Mod $\sigma^2_{H,\mathrm{M}}(\tau)$ exhibits random ripple due to poor confidence only at the very right end of the curve.

B. Equivalent Definition in the Frequency Domain

The MHVAR time-domain definition (5) can be translated to an equivalent expression in the frequency domain. In fact, it can be rewritten as the mean square value of the signal output by a linear filter, with impulse response properly shaped, receiving the input signal. In terms of $y(t)=x'(t)$, that is

$$\mathrm{Mod}\,\sigma^2_{\mathrm{H},M}(\tau)=\left\langle[y(t)*h_{\mathrm{MH}}(M,n,t)]^2\right\rangle \quad (7).$$

Hence, MHVAR can be equivalently defined in the frequency domain as the area under the PSD of the filter output

$$\mathrm{Mod}\,\sigma^2_{\mathrm{H},M}(\tau)=\int_0^\infty S_y(f)\left|H_{\mathrm{MH}}(M,n,f)\right|^2 df \quad (8)$$

where $S_y(f)$ is the one-sided PSD of $y(t)$ and thus $S_y(f) = S_x(f)\cdot(2\pi f)^2$. The square magnitude $|H_{\mathrm{MH}}|^2$ takes the asymptotic expression, for $n\to\infty$ and keeping $n\tau_0=\tau$ constant:

$$\lim_{\substack{n\to\infty\\ n\tau_0=\tau}}\left|H_{\mathrm{MH}}(M,n,f)\right|^2=\frac{2^{2(M-1)}}{M!}\left(\frac{\sin\pi\tau f}{\pi\tau f}\right)^4(\sin\pi\tau f)^{2(M-1)} \quad (9).$$

Extrapolating the behaviour of the MAVAR transfer function (M=2) [7][8], we infer that this limit is approached quickly for fairly low values of n (few units). The square magnitude (9) is plotted in Fig. 1 for some values of M, having omitted the constant factor $2^{2(M-1)}/M!$ and normalized the Fourier frequency f to the inverse $1/\tau$ of the observation interval.

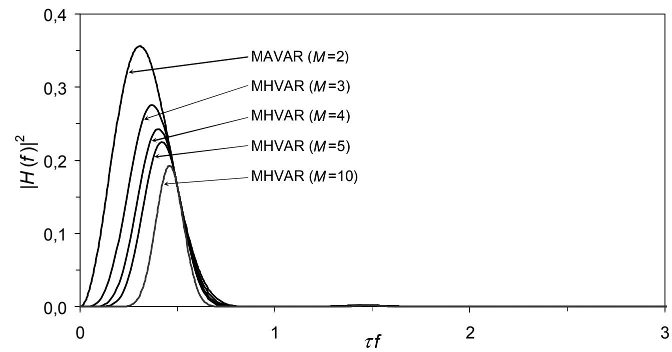

Fig. 1: Square magnitude of MHVAR-M asymptotic transfer functions.

Interesting enough is to notice that these transfer functions are pass-band, having magnitude shaped with a narrow main lobe centred at $f\cong 0.4/\tau$ and very low side lobes. Hence, MAVAR and MHVAR gather signal power selectively around a frequency proportional to $1/\tau$ (cf. eq. (8)), allowing high-resolution spectral analysis by computation over a range of τ.

Also, we notice that the main lobe of the transfer function becomes narrower by increasing M, i.e. variances become more selective in the frequency domain. Thus, MHVAR-M has even better spectral resolution than MAVAR. Nevertheless, it is difficult to evaluate analytically its actual improved ability in particular to estimate parameters of fractional noise.

C. Behaviour with Power-Law Random Signals

It is convenient to generalize the LRD power-law model of spectral density (3). As customary in characterization of phase and frequency noise of precision oscillators [14], we deal with random processes $x(t)$ whose one-sided PSD is modelled as

$$S_x(f)=\begin{cases}\displaystyle\sum_{i=1}^{P}h_{\alpha_i}f^{\alpha_i} & 0<f\le f_{\mathrm{h}}\\ 0 & f>f_{\mathrm{h}}\end{cases} \quad (10)$$

where P is the number of noise types considered, α_i and h_{α_i} are model parameters (α_i, $h_{\alpha_i}\in\Re$) and f_h is the upper cut-off frequency. Such random processes are commonly referred to as *power-law* or *fractional noise*.

Power-law noise with $-4\le\alpha_i\le 0$ has been revealed in practical measurements of various physical phenomena, such as phase noise of precision oscillators [8][12][14] and Internet traffic [1][2][6][7][13], whereas P should be not greater than few units for the model being useful. If the process $x(t)$ is simple LRD (3), then P=1 and $-1<\alpha_i<0$. Although values $\alpha_i\le -1$ yield model pathologies, such as infinite variance and nonstationarity, this model is commonly used, considering also that measurements have finite duration and bandwidth.

Under this general hypothesis of power-law PSD, first we notice that, since $|H_{\mathrm{MH}}(M,n,f)|^2$ in integral relationship (8) behaves as $\sim f^{2(M-1)}$ for $f\to 0$, MHVAR-M convergence is ensured for $\alpha_i > -1-2M$. Then, by considering separately each term of the sum in (10) and letting P=1, $\alpha=\alpha_i$, evaluation of

(8) yields corresponding time-domain functions Mod $\sigma^2_{H,\mathrm{M}}(\tau)$. Complete formulas for MAVAR (MHVAR-2) are reported in [8]. Moreover, Rutman [14] presents a detailed overview about recognizing power-law random noise and polynomial drifts in time-domain measures, including Allan/Hadamard variances.

In summary, with $x(t)$ model (10) and in the whole range of convergence $-1-2M < \alpha \le 0$, MHVAR-M is found to follow a simple power law for any M (ideally asymptotically for $n\to\infty$, $n\tau_0=\tau$, but in practice for $n>4$), i.e.

$$\mathrm{Mod}\,\sigma^2_{\mathrm{H},M}(\tau) \sim A_\mu \tau^\mu \qquad (11)$$

where $\mu = -3-\alpha$. If $P>1$, it is immediate to generalize (11) to summation of powers $\sum_i A_{\mu_i}\tau^{\mu_i}$.

This is a fundamental result. If $x(t)$ obeys (10), a log-log plot of Mod $\sigma^2_{H,\mathrm{M}}(\tau)$ looks ideally as a broken line made of P straight segments, whose slopes μ_i yield exponents $\alpha_i = -3-\mu_i$ of the fractional noise terms dominant in different ranges of τ.

D. Behaviour with Deterministic Signals

Here, the behaviour of MHVAR is studied when $x(t)$ includes offset, polynomial drifts, periodic signals and steps, which are major examples of nonstationarity in Internet traffic.

1) Offset and polynomial drift. Let $x(t)$ include an offset and polynomial drift, i.e. $x(t)=\sum_{j=0}^{M} C_j t^j$. By substitution in (5), we get that, as obvious, MHVAR is independent on data polynomial drift of order $<M$, but it reveals a $\sim t^M$ drift, then assuming trend $\sim\tau^{2M-2}$.

2) Periodic Signals. Let $y(t)=x'(t)$ be a sine wave at frequency f_m, i.e. $y(t)=A\sin 2\pi f_\mathrm{m} t$, with $S_y(f) = (A^2/2)\cdot\delta(f-f_\mathrm{m})$. Then, by substitution in (8)(9), we get (for $n\to\infty$, $n\tau_0=\tau$):

$$\mathrm{Mod}\,\sigma^2_{\mathrm{H},M}(\tau) = A^2\,\frac{2^{2(M-1)-1}}{M!}\left(\frac{\sin \pi f_m \tau}{\pi f_m \tau}\right)^4 \left(\sin \pi f_m \tau\right)^{2(M-1)} \quad (12).$$

Hence, MHVAR ripples with period $2/f_\mathrm{m}$.

3) Steps. Major examples of nonstationarity in Internet traffic traces are sudden changes of the average bit rate, due for instance to traffic rerouting or link capacity adjustment. Our simulation results (Fig. 4) show that the actual impact on MHVAR of an input step superposed to fractional noise is significant only if the step amplitude is very high. However, steps in input data affect MHVAR slope (with M even) less than LD.

V. Using MHVAR for Estimating the Hurst Parameter

Let us consider a LRD process with PSD (3) characterized by Hurst parameter $1/2 < H < 1$. Then, from (4)(11), for any M Mod $\sigma^2_{H,\mathrm{M}}(\tau)$ follows $\sim\tau^\mu$ (ideally for $n\to\infty$) with exponent $\mu = 2H-4$. In brief, it is possible to estimate the Hurst parameter of a sample realization $\{x_k\}$, supposed with PSD (3), by the following procedure adapted from that of MAVAR [6][7]:

1) compute Mod $\sigma^2_{H,\mathrm{M}}(\tau)$ by estimator (6), based on the data sequence $\{x_k\}$ for increasing integer values $1 \le n < N/(M+1)$ (we use a geometric progression of ratio 1.1);
2) estimate its average slope μ in a log-log plot for $n>4$ and excluding also highest values of n, where confidence is lowest, by best fitting a straight line to the curve (e.g., by least square error);
3) if $-3 < \mu < -2$ (i.e., $-1 < \alpha < 0$, $0 < \gamma < 1$), get the estimate of the Hurst parameter as

$$H = \mu/2 + 2 \qquad (13).$$

Under the more general hypothesis of power-law PSD (10), as noticed in Sec. IV.C, then up to P slopes μ_i can be estimated ($-3 \le \mu_i < 2M-2$) to yield the exponents $\alpha_i = -3-\mu_i$ ($-1-2M < \alpha_i \le 0$) of the f^{α_i} noise components prevailing in different ranges of τ.

Finally, some care should be exercised against the presence in data analyzed of deterministic components (e.g., big steps), which cause trends in Mod $\sigma^2_{H,\mathrm{M}}(\tau)$ that may be erroneously ascribed to random power-law noise. On the other hand, polynomial drifts in the measured data are not a problem, unless their order is greater than M, which is very unlikely.

VI. Simulation Results

The validity and accuracy of the MHVAR method were evaluated by extensive simulations, comparing it to the well-established wavelet LD technique [2][4] and to the MAVAR method [6][7]. All LD results were computed using standard scripts [29] (Daubechies' wavelet with 3 vanishing moments).

A. Accuracy Evaluation

The LD, MAVAR and MHVAR (M=3) methods were applied to LRD pseudo-random data series $\{x_k\}$ of length N, generated with power-law one-sided PSD $S_x(f) = hf^\alpha$ ($-1 < \alpha \le 0$) for assigned values of $H = (1-\alpha)/2$. The generation algorithm is by Paxon [30]. In brief, it is based on spectral shaping: a vector of random complex samples, with mean amplitude equal to the square root of the desired value of $S_x(f_k)$ and phase uniformly distributed in $[0, 2\pi]$, is inversely Fourier-transformed to yield the time-domain sequence $\{x_k\}$.

First, 100 independent pseudo-random sequences $\{x_k\}$ of length $N = 131072$, with m_x=0 and variance σ_x^2=1, were generated for each of the 11 values $\{H_i\}$ = {0.50, 0.55, ..., 1.00}, corresponding to $\{\alpha_i\}$ = {0, -0.1, …, -1.0}. On the resulting 1100 time series, we applied the MHVAR-3, MAVAR and LD methods, getting three sets of estimates $\{\hat{H}_{i,j}\}$, for $i = 0, 1, \ldots, 10$ and $j = 1, 2, \ldots, 100$. We then evaluated the accuracy of these estimates compared to the assigned generation values H_i, calculating the estimation errors $\Delta_{i,j} = \hat{H}_{i,j} - H_i$. Furthermore, we compared the accuracy attained by the three methods on short sequences, when results are impaired by poor confidence. Thus, we repeated the same test as before, but on another set of 1100 sequences of length $N = 1024$.

Fig. 2 compares the absolute estimation errors $\{\Delta_{i,j}\}$ attained by the three methods on sequences of $N = 131072$ samples. For each value H_i, the mean $m_{\Delta i}$ and standard deviation $\sigma_{\Delta i}$, out of 100 estimation errors, are plotted.

Both MAVAR and MHVAR achieve better confidence than LD. Standard deviation of MHVAR estimates is on the average yet 20% smaller than that of MAVAR. Also, the mean of LD estimates departs significantly from the target H_i.

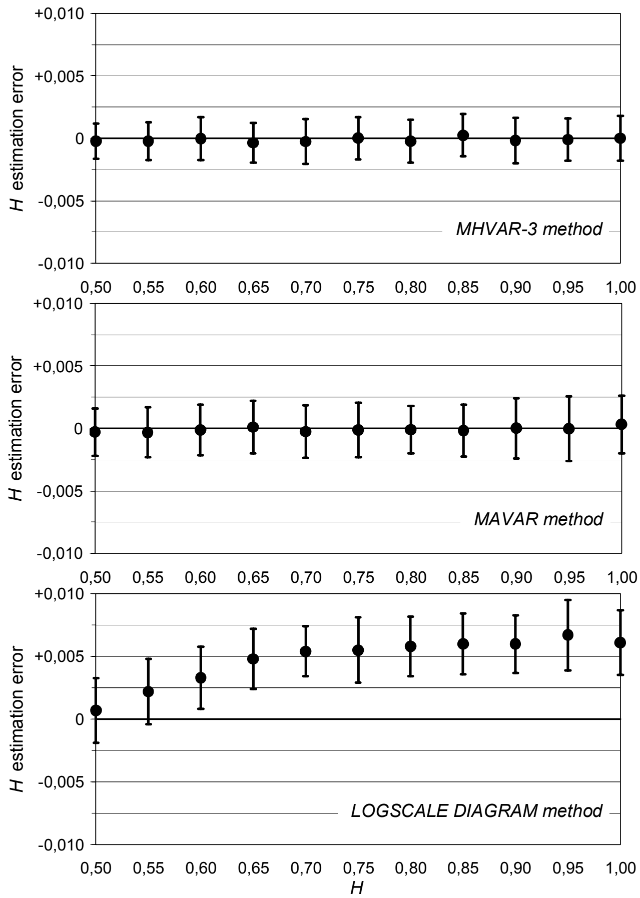

Fig. 2: Absolute estimation error of H attained by 3 methods (N=131072).

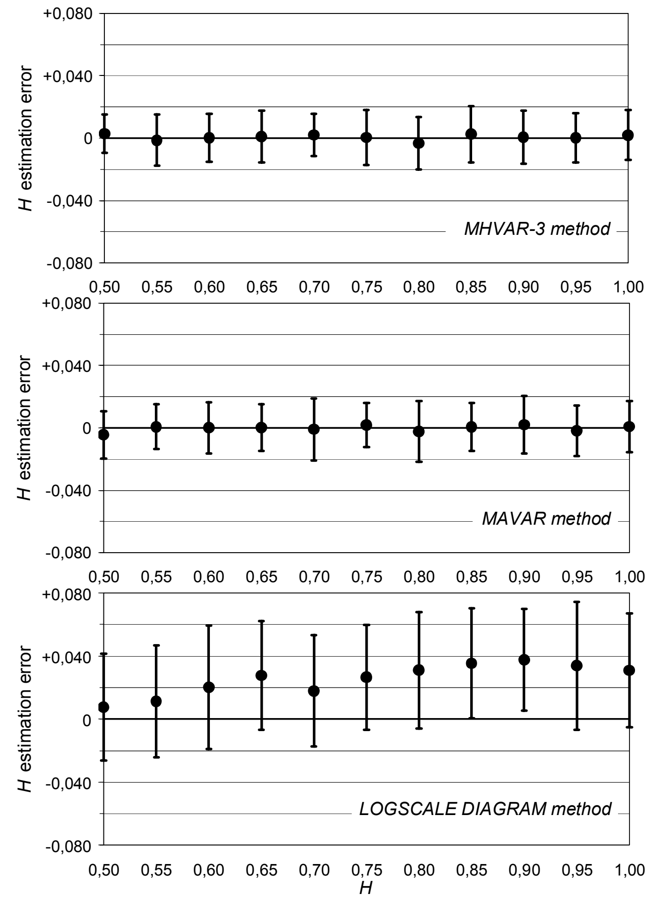

Fig. 3: Absolute estimation error of H attained by 3 methods (N=1024).

Similarly, Fig. 3 compares the estimation errors $\{\Delta_{i,j}\}$ on sequences of N=1024 samples. On such short sequences, MHVAR does not seem to perform better than MAVAR. Yet, both MAVAR and MHVAR achieve much better confidence than LD, which seems less efficient on short data sequences.

B. Impact of Steps Superposed to LRD Input Data

We evaluated MHVAR and LD on LRD data with steps superposed. Sequences of length $N = 1024$, 131072 were generated as $\{x_k\} = \{Au_{k-Q} + n_k\}$ ($k = 1, \ldots, N$), where $\{u_{k-Q}\}$ is the sampled unit step function $u(t)$ delayed Q time units ($1<Q<N$) and $\{n_k\}$ is a pseudo-random LRD series, with mean m_n=0 and variance σ^2_n=1, generated as before with PSD $S_n(f) = hf^{\alpha}$ for $\alpha = -0.60$ ($H = 0.80$). By varying Q and A, we found that:

- the step impact on MHVAR is maximum for $Q \cong N/2$;
- for M even, input steps affect MHVAR-M curves only at the right end, where the slope should not be considered anyhow due to poor confidence;
- for M odd, input steps do affect MHVAR-M, but with little impact on its slope;
- the step size A must be at least on the order of σ_n (i.e., very evident) to impact significantly MHVAR;
- input steps affect MHVAR-M (M even) less than LD.

Fig. 4 shows subsets of curves for N=131072, varying step size and delay as $0 \leq A \leq 2$ and $0<Q<N$. In comparing graphs, note that MHVAR is plotted over the full range $n < N/(M+1)$, whereas LD omits the last two scales (j>14).

C. Impact of the Difference Order M

We evaluated the impact of the order M on the MHVAR estimate accuracy, generating 100 independent pseudo-random sequences $\{x_k\}$ ($N = 1024$, 131072) as in Sec. VI.A for each of the 11 values $\{H_i\} = \{0.50, \ldots, 1.00\}$. Fig. 5 plots, for $2 \leq M \leq 10$ and $N = 1024$, 131072, the mean of the 11 mean values $m_{\Delta i}$ and of the 11 standard deviations $\sigma_{\Delta i}$ (i=0, …, 10).

First, we point out that the mean error is virtually 0 and with no bias. Moreover, these results confirm the confidence improvement of MHVAR-3 compared to MAVAR for N=131072: the mean $\sigma_{\Delta i}$ of MHVAR-3 is 25% smaller (cf. Fig. 2). This confidence gain is significant, since this figure is computed over 1100 independent estimates.

Conversely, we notice that increasing the order M>4 does not improve confidence further for N=131072, whereas it even worsen it for N=1024. This behaviour on short series is explained considering that estimator (6) averages N-$(M+1)n$+1 terms, each embracing $(M+1)n$ samples, and thus suffers little confidence when $(M+1)n$ approaches N.

In general, the confidence seems not to be improved by increasing the MHVAR order M>4, although $H_{\mathrm{MH}}(f)$ becomes more selective (cf. Fig. 1). Actually, better spectral *resolution* does not mean necessarily more accurate estimation of the $1/f^{\alpha}$ spectrum slope, which decays uniformly.

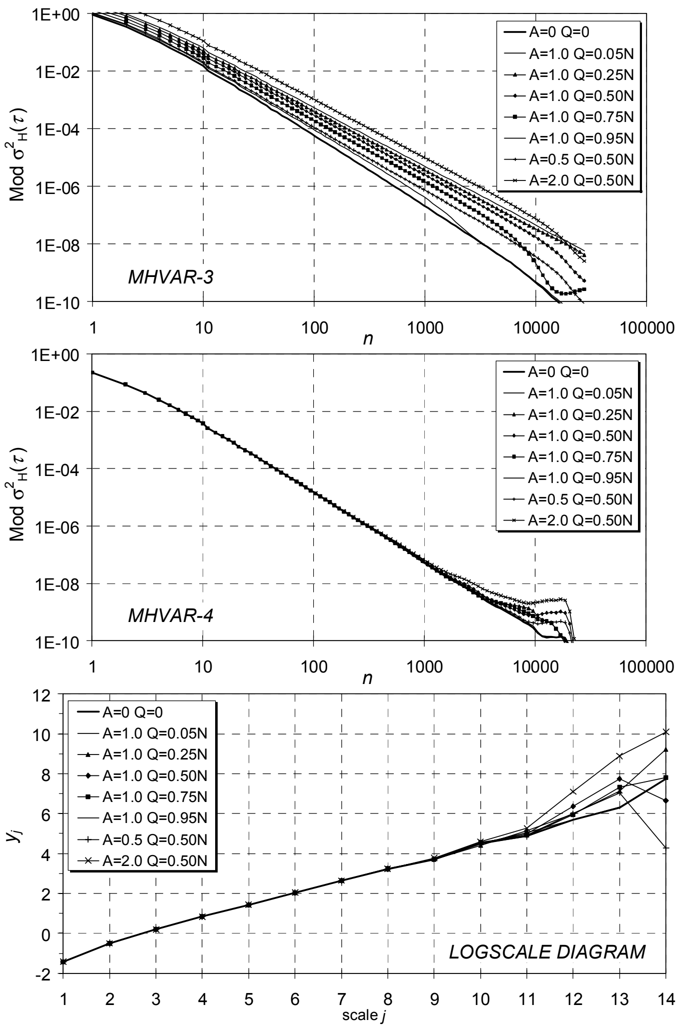

Fig. 4: MHVAR-3, MHVAR-4 and LD computed on pseudo-random LRD sequences $\{n_k\}$ (N=131072, m_n=0, σ_n=1, H=0.80) with added step $\{Au_{k-Q}\}$.

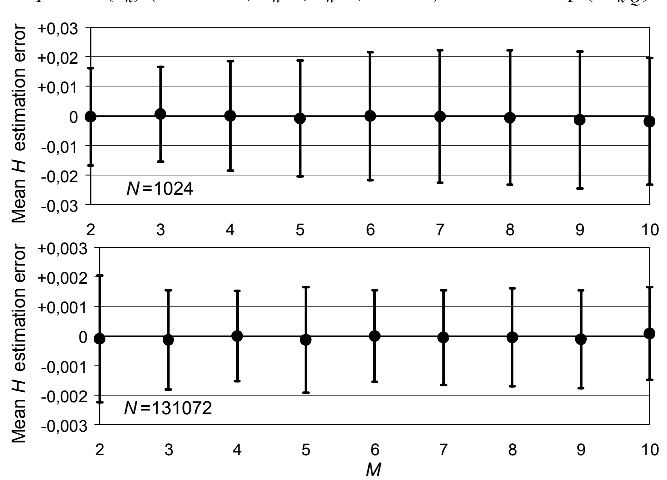

Fig. 5: Average mean E[$m_{\Delta i}$] and standard deviation E[$\sigma_{\Delta i}$] of the H estimation errors attained by the MHVAR-M method ($2 \le M \le 10$).

VII. Application to a Real IP Traffic Trace

We applied the MHVAR-3 and LD methods on a real IP traffic series [bytes/s] measured on a transoceanic link (MAWI

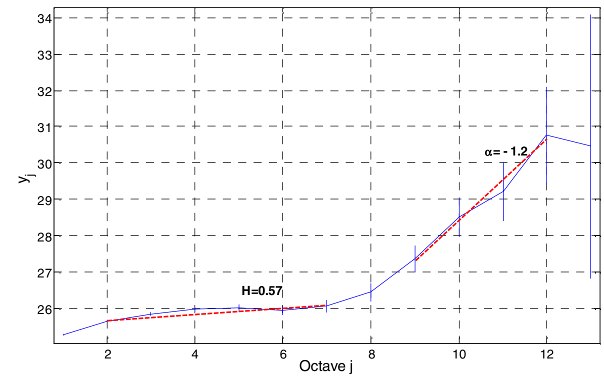

Fig. 6: Logscale diagram of a real IP bytes/time trace (MAWI Project [31], N=61600, τ_0=10 ms, T=616 s).

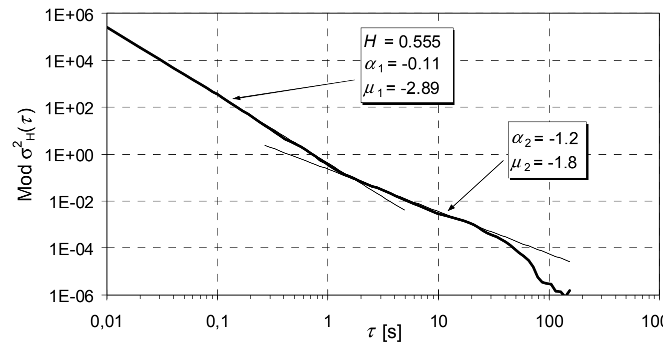

Fig. 7: Modified Hadamard Variance (M=3) of a real IP bytes/time trace (MAWI Project [31], N=61600, τ_0=10 ms, T=616 s).

[31]). The data series is made of N=61600 samples, acquired with sampling period τ_0=10 ms over a measurement interval T=616 s. No nonstationary trends, such as steps, are evident.

Figs. 6 and 7 show respectively the LD with 95%-confidence intervals and the MHVAR-3 (24 points/decade). We notice that the LD trend is more irregular (cf. the lower confidence of H estimates in Figs. 2 and 3), whereas MHVAR gives a clearer picture of the spectral characteristics of the sequence under analysis.

The MHVAR curve exhibits two regular slopes, namely μ_1 = -2.89 and μ_2 = -1.8. Almost no spurious ripples are visible in those intervals, in spite of the high density of points in which MHVAR has been computed.

Hence, two simple power-law (10) components are revealed by MHVAR: a main one with $\alpha_1 \cong$ -0.11 ($H \cong$ 0.555), dominant for 10 ms < τ < 2 s, and a secondary one with α_2 = -1.2, dominant for 2 s < τ < 20 s. Both estimates are in good agreement with slopes computed on the LD [29]. However, besides considering the multislope trend of LD, simulation results reported in this paper ensure that estimates obtained by MHVAR are more accurate and with better confidence.

VIII. Conclusions

In this paper, a Modified Hadamard Variance has been proposed for estimating the Hurst parameter H of LRD traffic series or, more generally, the exponent α of traffic series with $1/f^{\alpha}$ power-law spectrum. So far, this variance has been given little attention in literature. Thus, MHVAR definition and

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properties more relevant to this aim have been studied. The behaviour of MHVAR with power-law random processes and some common deterministic signals has been investigated.

The H estimation accuracy of MHVAR was evaluated on LRD pseudo-random sequences, by comparison to the well-known wavelet LD technique and to the MAVAR method [6][7]. Our extensive simulations showed that MHVAR, among the various variances investigated, exhibits the highest accuracy and confidence in fractional-noise parameter estimation, even slightly better than MAVAR, at the cost of some heavier computational effort, still affordable in most cases.

Moreover, besides its excellent spectral resolution capabilities, MHVAR has the advantage of converging also on very-low frequency fractional noise (e.g., with PSD $\sim f^5$ or $\sim f^6$) as well as being applicable on data including polynomial drift. Also, MHVAR proved quite robust against steps in input data: the actual impact of steps on Mod $\sigma^2_{H,\mathrm{M}}(\tau)$ is limited or negligible in most practical cases.

Finally, MHVAR was applied on a real IP traffic trace. Compared to LD, MHVAR gave a clearer spectral characterization of the traffic series analyzed. Two simple power-law noise components were revealed, with PSD $k_1/f^{0.11} + k_2/f^{1.2}$. The first term, dominant for $\tau < 2$ s, is LRD with $H \cong 0.555$.

In conclusion, we point out that MHVAR is not proposed as ultimate tool for traffic analysis. Rather, we believe that it may complement usefully other established techniques, e.g. MAVAR and LD, due to several advantages. Among them, we highlight in particular:

1) excellent spectral resolution (cf. Fig. 1);
2) efficient use of input data, yielding excellent confidence in parameter estimation of power-law processes (10) (e.g., LRD), even slightly better than MAVAR (cf. Figs. 2 and 3);
3) convergence to finite values for all types of power-law processes (10) with $\alpha > -1-2M$ ($\alpha_i \in \Re$);
4) insensitivity to polynomial drifts of order up to M-1;
5) robustness against various other common nonstationary (deterministic) components in data analyzed (e.g., steps).

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