Clock Stability Measure Dependence on Time Error Sampling Period

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Abstract

The introduction of SDH based networks rises new important synchronization issues to be carefully investigated. In particular, the telecommunication standard bodies are mainly considering, for the specification of clocks, five frequency stability quantities: Allan Variance, Modified Allan Variance, Time Variance, root mean square of the Time Interval Error and Maximum Time Interval Error. In this paper, the influence of Time Error measurement sampling period on the behaviour of these quantities is analyzed. Computer simulation and experimental measurement results are reported, with the aim of providing useful information for specification design. The results obtained show that the Modified Allan Variance and Time Variance behaviour is substantially dependent on the chosen measurement sampling period in the observation interval regions where white phase noise dominates.

1. Introduction

The introduction of Synchronous Digital Hierarchy (SDH) [1] technology in telecommunication networks rises new important issues related to synchronization, which have to be carefully investigated in order to fully exploit SDH capabilities. In particular, one of the most important open issues, both in academia and in the standardization bodies, is the identification of suitable quantities for characterizing time and frequency stability of timing signals in telecommunication networks.

At present, five quantities are mainly considered for the specification of timing interfaces requirements [2]: the *Allan Deviation* (ADEV), the *Modified Allan Deviation* (MADEV), the *Time Deviation* (TDEV), the *root mean square of Time Interval Error* (TIErms), and the *Maximum Time Interval Error* (MTIE) [3][4][5]. These quantities differ in their ability [6][7] of characterizing the most common noises affecting timing signals generated by actual clocks or distributed via a telecommunication network, and in their usefulness in designing synchronization systems. In the near future, indeed, the exigence of performing stability measurements on timing signals will be more and more stringent in order to design and verify the timing performance both of a single network equipment and of the overall synchronization network.

The practical measurement of all the above mentioned quantities, according to the last directions in current standards [2][8][9], is based on the acquisition of equally spaced samples of the Time Error (TE) between the clock under test and a reference clock. Due to practical constraints in the acquisition and in the storing of TE data, some preliminary crucial choices must be done before performing the stability measurements: in particular, the choice of the TE sampling period implies a trade-off between the resolution and the duration of the measurements, as it will be shown later.

In this paper, the influence of the measurement sampling period on the behaviour of the five stability quantities, in the presence of all common clock noises, is analyzed, with the aim of providing useful information for verifying standard specification requirements. Firstly, the so called power-law characterization of noises and the standard estimators of the stability quantities are introduced. Then, utilizing a computer simulation approach, results showing the influence of the measurement sampling period on the stability quantities are reported. Moreover, experimental measurement results on actual clocks are provided, in order to verify the main conclusions drawn by computer simulation and point out the impact that the sampling period can have in practical measurements.

2. Clock Noise Model

In telecommunications, a *clock* is a device able to supply a timing signal, ideally periodic, usable for the control of telecommunication systems. A mathematical model describing an actual timing signal $s(t)$ is given by [3][10]

$$s(t) = A \sin \Phi(t) \tag{1}$$

where A is a constant amplitude coefficient and $\Phi(t)$ is the *total instantaneous phase* expressed by

$$\Phi(t) = 2\pi(\nu_{\text{nom}} + \Delta\nu)t + \pi D \nu_{\text{nom}} t^2 + \varphi(t) + \Phi_0 \tag{2}$$

where $\Delta\nu$ represents the *frequency offset* of the actual clock from the *nominal frequency* ν_{nom}, D is the *linear fractional frequency drift* rate, mainly describing oscillator ageing effects, $\varphi(t)$ is the *random phase deviation*, modelling oscillator intrinsic phase noise sources, and Φ_0 is the initial phase offset.

The generated *Time function* $T(t)$ of a clock is defined, in terms of its total instantaneous phase, as

$$T(t) = \frac{\Phi(t)}{2\pi\nu_{\text{nom}}} \tag{3}$$

It is worthwhile noticing that for an ideal clock $T_{id}(t)=t$ holds, as expected. For a given clock, the *Time Error function* $x(t)$, between its time $T(t)$ and a reference time $T_{ref}(t)$, is defined as

$$x(t) = T(t) - T_{ref}(t) \tag{4}$$

In the frequency domain, the model most frequently used to represent clock phase noise is the so-called *power law model* [10], in which the *Power Spectral Density* (PSD) $S_\varphi(f)$ of $\varphi(t)$ is described by a sum of terms, each varying as an integer power of the Fourier frequency

$$S_\varphi(f) = \begin{cases} \sum_{\alpha=n_1}^{n_2} m_\alpha f^\alpha & 0 \le f \le f_h \\ 0 & f > f_h \end{cases} \tag{5}$$

where n_1, n_2 and the m_αs are device dependent parameters, and f_h is an upper cut-off frequency.

The most common noise types which dominate in precision oscillators are: White Phase Modulation (WPM) for α=0, Flicker Phase Modulation (FPM) for α=-1, White Frequency Modulation (WFM) for α=-2, Flicker Frequency Modulation (FFM) for α=-3, and Random Walk Frequency Modulation (RWFM) for α=-4. All the frequency stability quantities are sensitive, with different ability, to the presence of these noises on a timing signal [6][7].

3. Standard Estimators of Frequency Stability Quantities

According to ITU-T standards, frequency stability assessment of clocks in SDH networks is based on time domain measurement of the TE process $x(t)$ between the timing signal generated by the Clock Under Test (CUT) and a reference clock. Sequences of TE samples $\{x_i\}$ defined as

$$x_i = x(t_0 + (i-1)\tau_0) \qquad i = 1,2,\ldots,N \tag{6}$$

where t_0 is the initial observation time and τ_0 is the sampling period, are measured using digital counters and stored for numerical post-processing. The samples x_i are typically measured between two corresponding zero-crossing of the timing signals generated by the CUT and the reference clock, as shown in fig. 1.

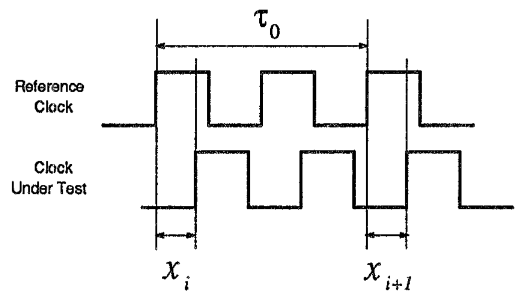

Fig. 1: Time Error measurement and sampling

For the five stability quantities considered in this paper, recent ITU-T standard documents [2] recommend the following estimators:

$$\mathrm{ADEV}(\tau) = \sqrt{\frac{1}{2n^2\tau_0^2(N-2n)}\sum_{i=1}^{N-2n}(x_{i+2n} - 2x_{i+n} + x_i)^2} \tag{7}$$

$$\mathrm{MADEV}(\tau) = \sqrt{\frac{1}{2n^4\tau_0^2(N-3n+1)}\sum_{i=1}^{N-3n+1}\left[\sum_{j=0}^{n-1}\left(x_{i+2n+j} - 2x_{i+n+j} + x_{i+j}\right)\right]^2} \tag{8}$$

$$\mathrm{TDEV}(\tau) = \sqrt{\frac{1}{6n^2(N-3n+1)}\sum_{i=1}^{N-3n+1}\left[\sum_{j=0}^{n-1}\left(x_{i+2n+j} - 2x_{i+n+j} + x_{i+j}\right)\right]^2} \tag{9}$$

$$\mathrm{TIErms}(\tau) = \sqrt{\frac{1}{N-n}\sum_{i=1}^{N-n}(x_{i+n} + x_i)^2} \tag{10}$$

$$\mathrm{MTIE}(\tau) = \max_{1\le k\le N-n}\left[\max_{k\le i\le k+n} x_i - \min_{k\le i\le k+n} x_i\right] \tag{11}$$

where $\tau = n\tau_0$ is the so called *observation interval* and $\lfloor z \rfloor$ denotes *"the greatest integer not exceeding z"*.

While for ADEV $n = 1,2,\ldots,\lfloor (N-1)/2 \rfloor$, for MADEV and TDEV $n = 1,2,\ldots,\lfloor N/3 \rfloor$, and for TIErms and MTIE $n = 1,2,\ldots,N-1$.

Let us note that, for all the above quantities, the sampling period τ_0 constraints both the minimum and the maximum values of the observation interval τ: the shorter τ_0 the shorter is the minimum observation interval, and viceversa, for a fixed N. In practical applications, the observation interval τ can range from 10^{-3} s to 10^6 s. With the aim at obtaining an overall behaviour of such quantities on the whole range of τ, one should set $\tau_0 = 10^{-3}$ s, thus increasing enormously both the need of storage capability and the computation time (e.g., for TDEV one should collect in real time N=$3\cdot10^9$ samples x_i).

In order to overcome such problems, one might operate on subranges of the whole range as follows:

1) for each subrange, choose τ_0 equal to the minimum observation interval of the subrange and collect measurement data;
2) perform, for each subrange, the calculation of the stability quantities;
3) *juxtapose* the resulting curves.

To be sure of the fairness of such an approach, an analysis on the influence of τ_0 on the five mentioned stability quantities was carried out. In the following sections, the main results obtained utilizing a computer simulation approach are shown, together with some experimental measurement results on actual clocks.

4. Computer Simulation Results

In order to simulate WPM noise, firstly two uniform pseudo-random sequences of length N=4096 were generated. Then, applying a well known transformation formula [11], a white Gaussian pseudo-random sequence was obtained, thus approximating a WPM noise. Finally, spectral shaping was accomplished by filtering the WPM noise sequence through a 1/2 order integrator [12] with transfer function $\mathrm{H}_{1/2}(f) = 1/\sqrt{j2\pi f}$, to generate the FPM noise sequence. Repeatedly filtering through $\mathrm{H}_{1/2}(f)$ yielded WFM, FFM, RWFM noise sequences (see eqn. (5)).

According to standard estimators, ADEV, MADEV, TDEV, TIErms and MTIE were evaluated starting from each of these five noise sequences. Then, the same calculations were carried out on sequences obtained by repeated decimation of the above five sequences, yielding sequences with length $N_1 = N/2$, $N_2 = N/4$, $N_3 = N/8$. This approach is clearly equivalent to sampling the same segment of a given noise process with the sampling periods τ_0, $2\tau_0$, $4\tau_0$, $8\tau_0$.

In fig. 2, 3 and 4 the computed curves of ADEV, TDEV and MTIE for each noise type (viz. WPM, FPM, WFM, FFM, RWFM) and for different sampling periods τ_0, $2\tau_0$, $4\tau_0$, $8\tau_0$ (corresponding to N=4096, N_1=2048, N_2=1024, N_3=512, respectively) are depicted. MADEV results are not reported, since the relationship

$$\mathrm{TDEV}(\tau) = \frac{\tau}{\sqrt{3}}\mathrm{MADEV}(\tau) \tag{12}$$

holds. Furthermore, in fig. 5, the computed curves of TIErms are depicted only for WPM, FPM and WFM noises, since for the FFM and RWFM noises TIErms does not theretically converge.

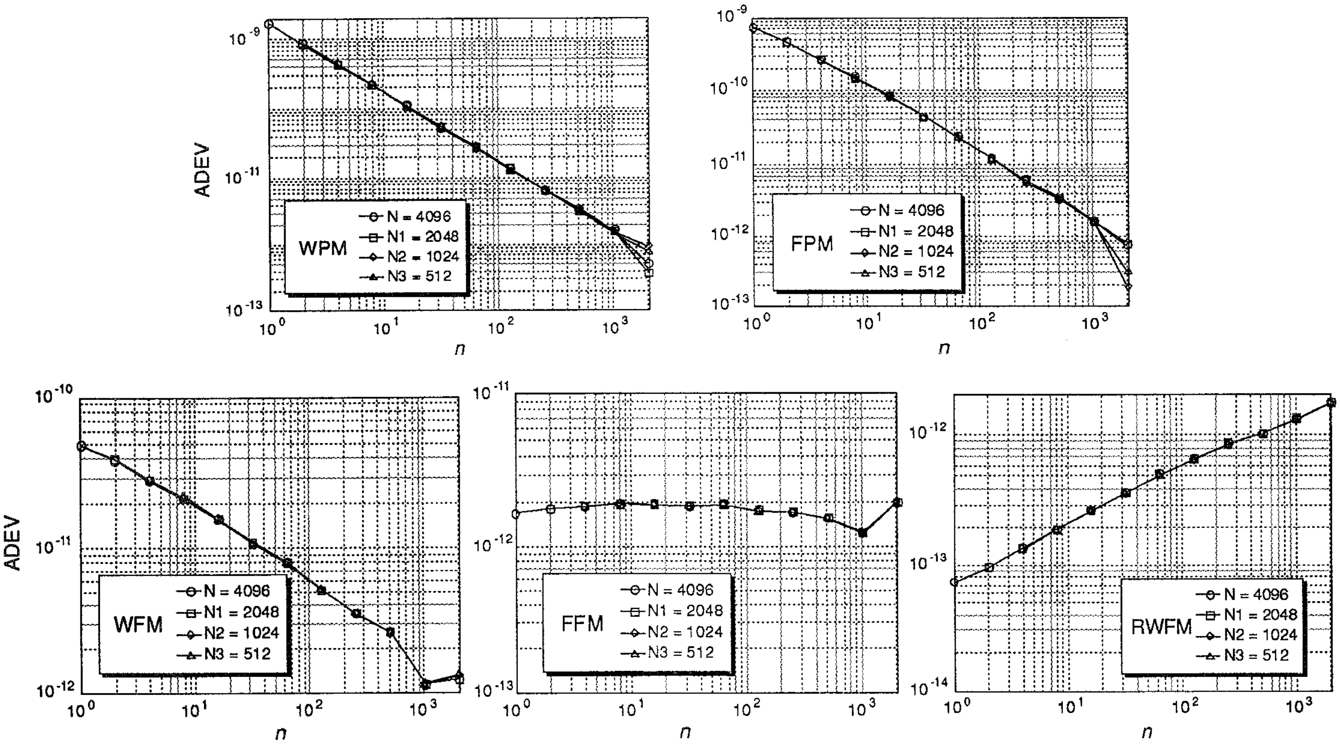

Fig. 2:
ADEV($n\tau_0$) results for WPM, FPM, WFM, FFM, RWFM simulated noises and for different sampling periods $\tau_0, 2\tau_0, 4\tau_0, 8\tau$
(corresponding to N=4096, N_1=2048, N_2=1024, N_3=512)

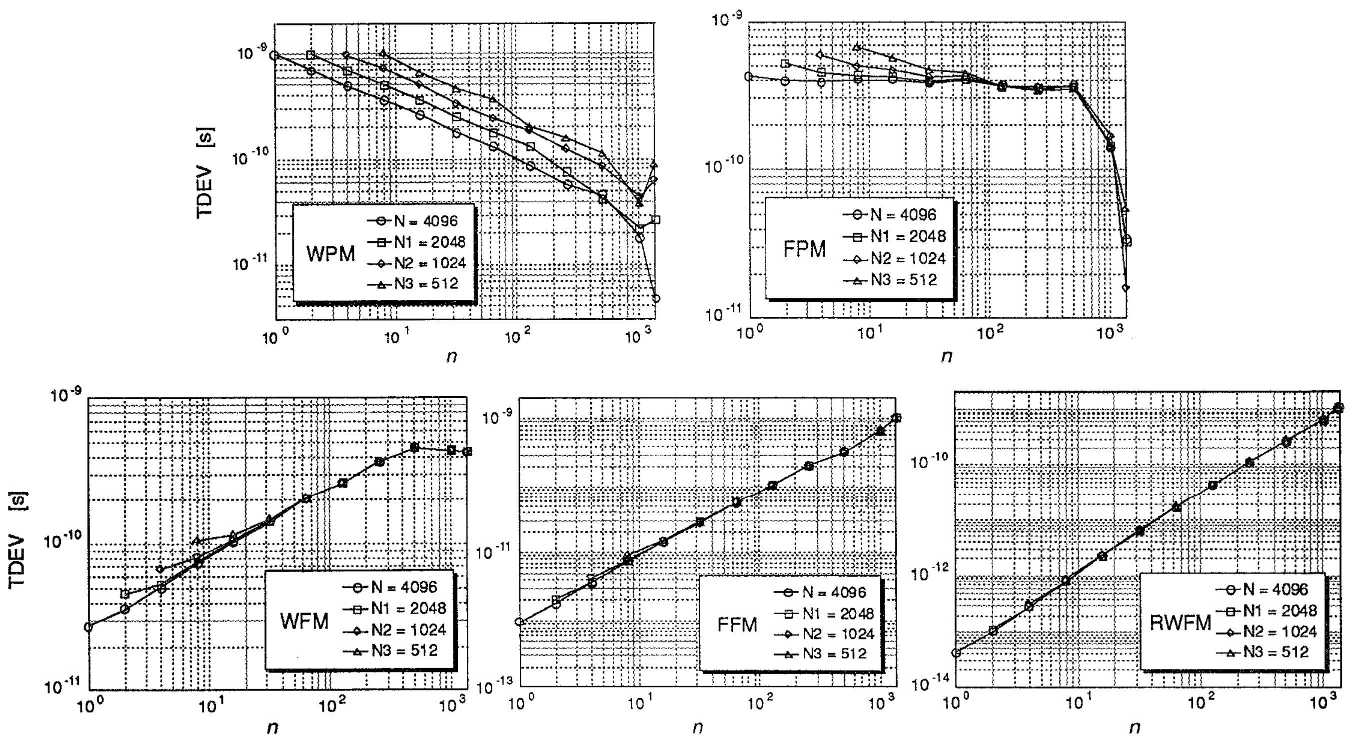

Fig. 3:
TDEV($n\tau_0$) results for WPM, FPM, WFM, FFM, RWFM simulated noises and for different sampling periods $\tau_0, 2\tau_0, 4\tau_0, 8\tau_0$
(corresponding to N=4096, N_1=2048, N_2=1024, N_3=512)

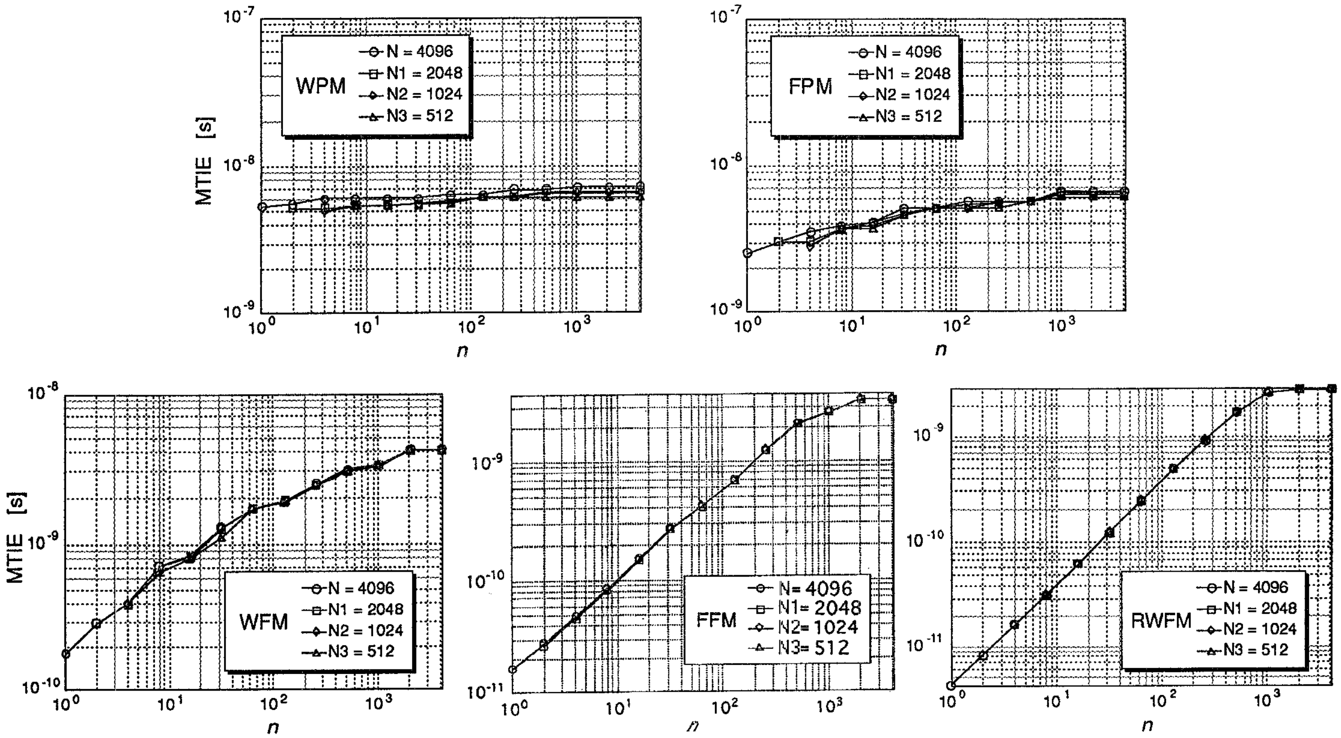

Fig. 4:
MTIE($n\tau_0$) results for WPM, FPM, WFM, FFM, RWFM simulated noises and for different sampling periods $\tau_0, 2\tau_0, 4\tau_0, 8\tau$
(corresponding to N=4096, N_1=2048, N_2=1024, N_3=512)

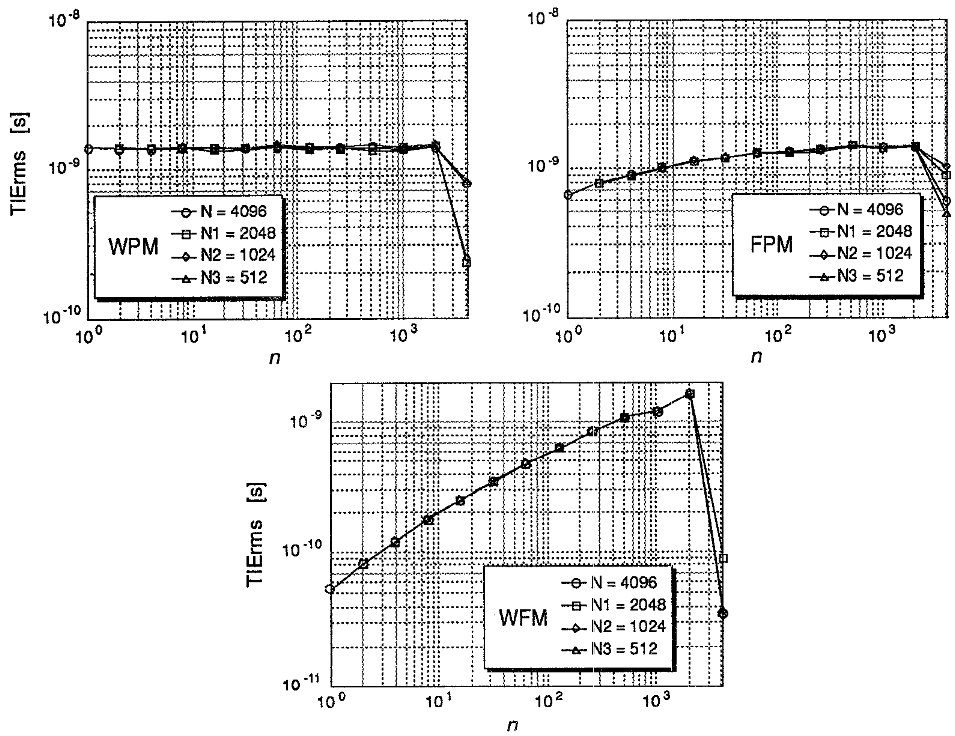

Fig. 5:
TIErms($n\tau_0$) results for WPM, FPM, WFM simulated noises and for different sampling periods $\tau_0, 2\tau_0, 4\tau_0, 8\tau_0$
(corresponding to N=4096, N_1=2048, N_2=1024, N_3=512)

By inspection of the presented curves, one observes that ADEV, MTIE and TIErms quantities do not show any significatively different behaviour as the sampling period is varied, for all the noise types considered. Conversely, MADEV and TDEV quantities exhibit substantial quantitative differences in the presence of WPM noise. Finally, let us also remark the poor statistical confidence in the ending right part of the curves, where the stability quantities are calculated averaging a too small number of samples.

5. Measurement Results

Experimental measurements, based on a high performance time counter [13], were carried out on the synchronization unit of a Public Switched Telephone Network (PSTN) digital exchange. The *synchronized clock measurement configuration* [2] was adopted: the timing signal under test and the reference one (see fig. 1) were respectively the output and the input of the synchronization unit operating in the slave mode. In such a configuration, the sequence $\{x_i\}$ reflects internal clock random noises only, and, hence, is unaffected by any deterministic component (e.g., frequency offset and drift). Both signals were 2.048 Mbit/s HDB3 coded.

A sequence $\{x_i\}$ of N=250000 TE samples, with sampling period τ_0=15 ms, was taken and, as done in the simulation approach, samples decimation was accomplished, yielding other two sequences of N_1=25000 and N_2=2500 samples, with sampling periods $10\tau_0$=150 ms and $100\tau_0$=1.5 s respectively.

From these three sequences, the stability quantities were evaluated. The measurement results confirm the conclusions stemming from simulations. As an example of the these measurement results, in fig. 6 the TDEV curves are reported: the slopes of the TDEV curves show the dominant presence on the timing signal under test of WPM and FFM noises. In particular, note that the WPM noise appears to be dominant in the range $10^{-2}\text{s}\leq\tau\leq 10\text{s}$, and that, at any given observation interval in that region, TDEV takes increasing values as the sampling period is increased. No such a dependence is recognizable in the range $10\text{s}\leq\tau\leq 10^3\text{s}$, where FFM noise dominates.

6. Conclusions

In this paper, the influence of the measurement sampling period on the behaviour of five frequency stability quantities (viz. ADEV, MADEV, TDEV, TIErms, MTIE) was analyzed, in the presence of all common clock noises, with the aim of providing useful information for specification design. Computer simulation and experimental measurement results on actual clocks were provided, in order to point out the impact that a different measurement sampling period τ_0 can have in practical cases.

The results shown exhibit significant dependence on τ_0 only for the MADEV and TDEV quantities, especially when WPM and FPM noises are present. This dependence must be taken into account when the approach of τ axis partitioning described in section 3 is used. Moreover, clock standards based on TDEV masks should always include the specification of the sampling period τ_0 used.

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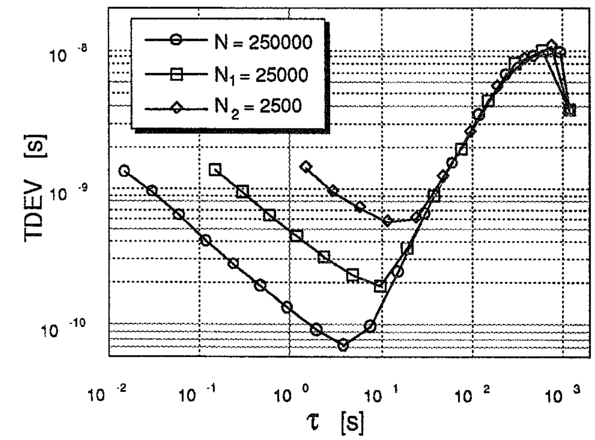

Fig. 6:
TDEV(τ) measurement results for different sampling periods τ_0=15 ms, $10\tau_0$=150 ms, $100\tau_0$=1.5 s (corresponding to N=25000, N_1=25000, N_2=2500)

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