Compared Accuracy Evaluation of Estimators of Traffic Long-Range Dependence

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***Abstract* — Internet traffic exhibits self-similarity and long-range dependence (LRD). Accurate estimation of statistical parameters characterizing self-similarity and LRD is an important issue, aiming at best modelling traffic e.g. to the purpose of network simulation. Major attention has been devoted to designing algorithms for estimating the Hurst parameter H of LRD traffic series or, more generally, the exponent $\gamma \geq 0$ of data with $1/f^{\gamma}$ power-law spectrum. In this paper, by evaluation on thousands of pseudo-random LRD data series, we compare the H and γ estimation accuracy attained by some of the most widely used methods mentioned above: variance-time plot, R/S statistic, lag-1 autocorrelation, wavelet logscale diagram, Modified Allan and Hadamard Variances. In literature, there are almost no detailed comparison studies on the actual accuracy attained by various methods. Thus, our detailed results will be valuable for those interested to the analysis of traffic or, in general, of power-law data.**

***Index Terms* — Communication traffic, Internet, long-range dependence, time domain analysis, wavelet transforms, traffic measurement (communication).**

I. Introduction

Packet traffic exhibits intriguing temporal correlation properties, such as self-similarity and long memory (long-range dependence) [1][2]. Contrary to the classical Poisson assumption, these properties emphasize long-range time-correlation between packet arrivals. Fractional noise and fractional Brownian motion models are often used to describe such behaviour of Internet traffic series, which include, but are not limited to, cumulative or incremental data count transmitted over time, inter-arrival time series of successive TCP connections or IP packets, etc.

In a self-similar random process, a dilated portion of a realization has the same statistical characterization than the whole. "Dilating" is applied on both amplitude and time axes of the sample path, according to a scaling parameter H called Hurst parameter. On the other hand, long-range dependence (LRD) is a long-memory property observed on large time scales, usually equated to an asymptotic power-law decrease of the power spectral density (PSD) $\sim f^{-\gamma}$ ($\gamma \geq 0$) or, equivalently, of the autocovariance function. Under some hypotheses, the integral of a LRD process is self-similar with H related to γ (e.g., fractional Brownian motion, integral of fractional Gaussian noise).

Accurate estimation of statistical parameters that characterize self-similarity and LRD is an important topic that has been studied in several works, aiming at best modelling of traffic for example to the purpose of network simulation. Major attention has been devoted to designing algorithms for estimating parameters H and γ of data sequences supposed LRD.

Among various techniques proposed in literature to this aim, the direct approach consists in analysing data in the frequency domain: a simple *periodogram* log-log plot yields a straightforward estimation of γ and H from its slope [3][4]. Nevertheless, it should be noted that this technique is not well suited to analyze processes with power-law spectrum $\sim f^{-\gamma}$ ($\gamma > 0$), which gathers most power for $f \to 0$, being standard periodogram samples evenly spaced in frequency.

For such random processes, time-domain analysis is better suited. For example, the basic *Variance-Time (VT) plot* method [2][3][5][6] studies the variance of aggregated time series, made of samples computed by averaging non-overlapping data windows. By observing the decay of the variance plot, as a function of the window width, γ and H can be estimated.

Many other time-domain quantities have been proposed to estimate parameters of LRD data sequences: more or less sophisticated, they are all based on some kind of data averaging in the time domain over a variable observation interval. These quantities, log-log plotted versus the observation interval, exhibit a regular slope on γ and H if computed on LRD data. Among the various time-domain methods, the *rescaled adjusted range statistic* (R/S statistic) [3][7][8] and the *Allan Variance* (AVAR) [9]–[12] are among most interesting ones.

Yet another time-domain method was proposed for quick identification of power-law noise with integer exponent: *lag-1 autocorrelation* [13], consisting of evaluating data autocorrelation $R(k)$ at lag $k = 1$. This method may be also adapted, with some caution, for approximate estimation of fractional parameters γ and H.

Beyond the time and frequency domains dichotomy, a breakthrough occurred when techniques based on wavelet analysis were introduced for fractional noise estimation [1][14]-[17]. Due to their sensitivity to scaling phenomena over a range of scales, wavelets are well suited to detect self-similarity or other more complex scaling behaviours. Among wavelet-based techniques, the so-called Daubechies wavelet *Logscale Diagram* (LD) is of utmost importance [16].

In a different context, the *Modified Allan Variance* (MAVAR) is a well known time-domain quantity, originally proposed (1981) for frequency stability characterization of precision oscillators [12][18]–[21], purposely designed to discriminate noise types with power-law spectrum. Telecommu-

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nications standards (ANSI, ETSI, ITU-T) specify some network synchronization requirements in terms of Time Variance (TVAR), closely related to MAVAR [22]. More recently, MAVAR was also proposed as LRD traffic analysis tool, pointing out its superior accuracy in estimating H and γ [23]–[26].

A *Modified Hadamard Variance* (MHVAR) was also proposed [24], by generalizing the concept of MAVAR to higher-order differences of data analogously to the classic unmodified Hadamard Variance (HVAR) [11][27]—[29]. Both MAVAR and MHVAR were demonstrated to feature significantly better accuracy in estimating H and γ than the widely used LD [24].

In this paper, by evaluation on thousands of pseudo-random LRD data series, we compare the H and γ estimation accuracy attained by above methods: namely VT plot, R/S diagram, lag-1 autocorrelation, LD, MAVAR and MHVAR. In literature, there are almost no detailed comparison studies on the actual accuracy attained by various LRD estimators, with the notable exception [3]. Therefore, our results will be valuable to researchers involved in traffic analysis or, more generally, in power-law data estimation.

II. Self-Similarity and Long-Range Dependence

A random process $X(t)$ (say, cumulative packet arrivals in time interval $[0, t]$), is said to be *self-similar*, with scaling parameter of self-similarity or Hurst parameter $H>0$, $H\in\Re$, if

$$X(t) =_d a^{-H} X(at) \tag{1}$$

for all $a>0$, where $=_d$ denotes equality of all finite-order distributions [1]. In other terms, the statistical description of $X(t)$ does not change by *scaling* simultaneously its amplitude by a^{-H} and the time axis by a.

The class of self-similar (SS) processes is usually restricted to that of *self-similar processes with stationary increments* (SSSI), which are "integral" of a stationary process. For example, consider the δ-increment process of $X(t)$, defined as $Y_\delta(t) = X(t)-X(t-\delta)$ (say, packet arrivals in the last δ time units). For an SSSI process $X(t)$, $Y_\delta(t)$ is stationary and $0 < H < 1$ [1].

Long-range dependence (LRD) of a process is defined by an asymptotic power-law decrease of its autocovariance and PSD functions [1]. Let $Y(t)$ be a second-order stationary stochastic process. $Y(t)$ exhibits LRD if, equivalently, its autocovariance and two-sided PSD follow asymptotically

$$R_Y(\delta) \sim c_1 |\delta|^{\gamma-1} \quad \text{for } \delta \to +\infty,\ 0<\gamma<1 \tag{2}$$

$$S_Y(f) \sim c_2 |f|^{-\gamma} \quad \text{for } f \to 0,\ 0<\gamma<1 \tag{3}$$

In general, a random process with non-integer power-law PSD is also known as fractional (not necessarily Gaussian) noise. SSSI processes $X(t)$ with $1/2 < H < 1$ have LRD increments $Y(t)$, with

$$\gamma = 2H - 1 \tag{4}$$

Strictly speaking, H characterizes SS processes, but it is often used to label also the LRD increments of SSSI processes. In this paper, we follow this common custom: the expression "Hurst parameter of a LRD process" (characterized by γ) denotes actually the parameter $H = (\gamma+1)/2$ of its integral SSSI parent process.

III. Methods for Estimating LRD Parameters H and γ

For ease of understanding, this section recalls basic definitions of methods compared, viz. VT plot, R/S statistic, $R(1)$, LD, MAVAR and MHVAR. The input data sequence $\{x_i\}$ is assumed LRD and with length N. For all details and a precise treatise, the reader is referred to the bibliography cited.

A. Variance-Time Plot

The VT plot method [2][5][6] studies the variance $\sigma^2(m)$ of the aggregated sequence obtained by dividing the LRD series $\{x_i\}$ into non-overlapping blocks of length m and averaging them, i.e.

$$X_k^{(m)} = \frac{1}{m} \sum_{i=(k-1)m+1}^{km} x_i \tag{5}$$

for $k = 1, 2, \ldots, N/m$. The variance of the aggregated sequence can be estimated as the sample variance

$$\sigma^2(m) = \frac{1}{N/m} \sum_{k=1}^{N/m} \left[X_k^{(m)}\right]^2 - \left[\frac{1}{N/m} \sum_{k=1}^{N/m} X_k^{(m)}\right]^2 \tag{6}$$

For large N/m and m, this variance obeys the power law

$$\sigma^2(m) \sim m^{\gamma-1} \sigma_x^2 \tag{7}$$

Thus, by linear regression on a log-log plot of $\sigma^2(m)$ versus m, γ and H can be estimated.

B. Rescaled Adjusted Range Statistic (R/S)

The R/S statistic is one of the main time-domain methods for LRD estimation [3][7][8]. Based on the partial sum $Y(n) = \sum_1^n x_i$ and sample variance $S^2(n) = (1/n)\sum_1^n x_i^2 - Y^2(n)/n^2$, the R/S statistic is defined as

$$\frac{R(n)}{S(n)} = \frac{1}{S(n)} \left\{ \max_{0\le l\le n}\left[Y(l) - \frac{l}{n} Y(n)\right] - \min_{0\le l\le n}\left[Y(l) - \frac{l}{n} Y(n)\right] \right\} \tag{8}$$

Assuming LRD $\{x_i\}$ (3), for $n\to\infty$ its expected value follows

$$E\left[\frac{R(n)}{S(n)}\right] \sim C_H n^H \tag{9}$$

where C_H is a positive constant.

In practice, to estimate H using the R/S statistic, the input sequence $\{x_i\}$ is divided in K blocks. Then, for each lag n, R/S is computed at up to K starting points, taken evenly in different blocks. By linear regression on a log-log plot of $R(n)/S(n)$ versus n, H and γ can be estimated.

C. Lag-1 Autocorrelation

Recently, using lag-1 autocorrelation was proposed for quick identification of power-law noise [13]. The autocorrelation of sequence $\{x_i\}$ at lag k is estimated as

$$R(k) = \frac{\frac{1}{N}\sum_{i=1}^{N-k} (x_i - \bar{x})(x_{i+k} - \bar{x})}{\frac{1}{N}\sum_{i=1}^{N} (x_i - \bar{x})^2} \tag{10}$$

where $\bar{x} = (1/N)\sum_1^N x_i$ is the mean value. The lag-1 autocorrelation is simply the value $R(1)$ as given above. Based on this value, parameters γ and H can be estimated according to (2).

D. Daubechies Wavelet Logscale Diagram

Wavelet analysis has become very widely used to detect self-similarity and long-range dependence [1][14]-[17]. Among such techniques, the second-order LD is of utmost importance [16]: it analyzes data over an interval of scales j (octaves), ranging from 1 (finest detail) to a longest scale given by the series finite length. Assuming LRD data $\{x_i\}$ (3), this method is based on observing the asymptotical power-law behaviour of the wavelet detail variances across scales

$$E\left[d_x(j,k)^2\right] \sim C\,2^{j\gamma} \tag{11}$$

where $d_x(j,k)$ are the coefficients of Daubechies wavelets $\psi_{j,k}(t)$ in the decomposition of signal $x(t)$. These variances can be efficiently estimated as

$$\mu_j = \frac{1}{n_j}\sum_{k=1}^{n_j} d_x(j,k)^2 \tag{12}$$

where $n_j = 2^{-j}N$ is the number of coefficients available at octave j. The log-log plot of μ_j versus j is referred to as second-order LD. By linear regression, γ and H can be estimated (11).

E. Modified Allan and Hadamard Variances

The Modified Allan Variance (MAVAR) [12] and the Modified Hadamard Variance (MHVAR) [24] were demonstrated to feature outstanding accuracy in estimating H and γ [24].

Given a finite set of N samples $\{x_k\}$ spaced by τ_0 over a measurement interval T=(N-1)τ_0, MAVAR can be estimated as

$$\mathrm{Mod}\,\sigma_y^2(n\tau_0) = \frac{\displaystyle\sum_{j=1}^{N-3n+1}\left[\sum_{i=j}^{n+j-1}\left(x_{i+2n} - 2x_{i+n} + x_i\right)\right]^2}{2n^4\tau_0^2(N-3n+1)} \tag{13}$$

with n=1, 2,..., $\lfloor N/3\rfloor$. A recursive algorithm for fast computation exists [12], which cuts down the complexity of evaluating MAVAR for all $\lfloor N/3\rfloor$ values of n to O(N^2) instead of O(N^3).

On the other hand, MHVAR of order M can be estimated as

$$\mathrm{Mod}\,\sigma_{\mathrm{H},M}^2(\tau) = \frac{\displaystyle\sum_{i=1}^{N-(M+1)n+1}\left[\sum_{j=i}^{i+n-1}\sum_{k=0}^{M}\binom{M}{k}(-1)^k x_{j+kn}\right]^2}{M!\,n^4\tau_0^2\left[N-(M+1)n+1\right]} \tag{14}$$

with n = 1, 2, ..., $\lfloor N/(M+1)\rfloor$. MHVAR-M is based on the M^{th} difference of input data. MAVAR coincides with MHVAR for M=2. MHVAR and MAVAR differ from unmodified HVAR and AVAR, respectively, in the additional internal average over n adjacent samples: for n = 1 ($\tau=\tau_0$), they coincide. HVAR and MHVAR of order M=3 have been mostly considered in literature [11][27]—[29].

Assuming LRD data $\{x_i\}$ (3), both MAVAR and MHVAR-M are found to obey the simple power law (ideally asymptotically for $n\to\infty$, $n\tau_0=\tau$, but in practice for n>4)

$$\mathrm{Mod}\,\sigma_{\mathrm{H},M}^2(\tau) \sim A_\mu \tau^\mu \quad , \quad \mu = -3+\gamma \tag{15}$$

By linear regression on log-log plot, H and γ can be estimated.

IV. Estimation Accuracy Evaluation

The accuracy of the VT plot, R/S statistic, $R(1)$, LD-3, MAVAR and MHVAR-3 methods was evaluated and compared by extensive simulations. Estimators as outlined above and specified in the references cited were programmed. All LD results were computed running standard scripts [30], using Daubechies wavelet with three vanishing moments (LD-3).

Estimation methods were applied to LRD pseudo-random data series $\{x_k\}$ of length N, generated with one-sided PSD $S_x(f) = hf^{-\gamma}$ ($0 \le \gamma \le 1$) for assigned values of $H = (1+\gamma)/2$. The generation algorithm is by Paxson [31]: a vector of random complex samples, having amplitude equal to the square root of an exponentially-distributed variable with mean $S_x(f_k)$ and phase uniformly distributed in $[0, 2\pi]$, is inversely Fourier-transformed to yield the time-domain sequence $\{x_k\}$.

First, 100 independent pseudo-random sequences $\{x_k\}$ of length N = 131072, with mean m_x = 0 and variance σ_x^2 = 1, were generated for each of the 11 values $\{H_i\}$ = {0.50, 0.55, ..., 1.00}, corresponding to $\{\gamma_i\}$ = {0, 0.1, …, 1.0}. On the resulting 1100 time series, we applied all methods, getting six sets of estimates $\{\hat{H}_{i,j}\}$, for i = 0, 1, …, 10 and j = 1, 2, …, 100. Then, we evaluated the accuracy of these estimates, calculating the absolute estimation errors $\Delta_{i,j} = \hat{H}_{i,j} - H_i$.

We repeated the same test on another set of 1100 sequences of length N = 1024, to compare methods also on short sequences, where results are impaired by poor confidence.

Fig. 1 compares the estimation errors $\{\Delta_{i,j}\}$ attained by the six methods on sequences of N = 131072 samples. For each value H_i, the mean $m_{\Delta i}$ (dot), standard deviation $\pm\sigma_{\Delta i}$ (thick bar) and maximum-minimum excursion (thin bar), out of 100 estimation errors, are plotted. The max-min excursion is statistically not very meaningful, as it may include outliers, but it is plotted anyway, to show at a glance the estimation error maximum spreading. Similarly, Fig. 2 compares the estimation errors $\{\Delta_{i,j}\}$ obtained on short sequences of N = 1024 samples. In examining these compared plots, it should be considered that some have different scales on Y axes, due to the large difference of accuracy attained by the methods.

Finally, Fig. 3 plots the mean of the 11 mean values $m_{\Delta i}$ (dots) and of the 11 standard deviations $\sigma_{\Delta i}$ (bars) for N = 1024 and N = 131072, obtained by VT plot, R/S statistic, LD-3, MAVAR and MHVAR-3 methods ($R(1)$ results fall well out of scale). These results are averaged on 1100 samples (i.e., 100 pseudo-random sequences for each of 11 values H_i).

First, by inspection of Figs. 1, 2 and 3 and as already pointed out in [24], we notice that MAVAR and MHVAR-3 (plotted with same scale as LD-3) provide by far the most accurate estimates. In particular, MAVAR and MHVAR-3 are unbiased and achieve better confidence (i.e., smaller $\sigma_{\Delta i}$) than any other method. The accuracy of LD-3 approaches that of MAVAR and MHVAR-3, but only for long sequences. Estimates by VT plot and R/S statistic are significantly biased and

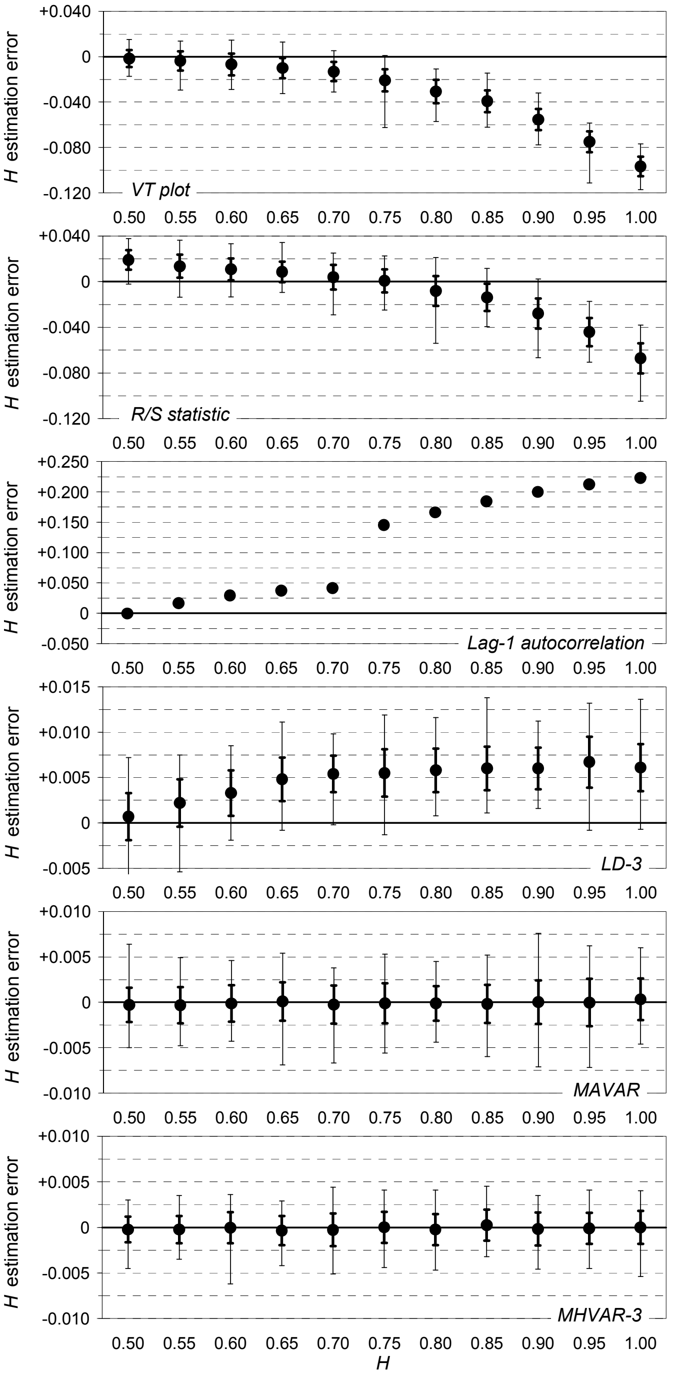

Fig. 1: Absolute estimation error of H attained by six methods (N=131072, mean, standard deviation and min-max excursion out of 100 samples).

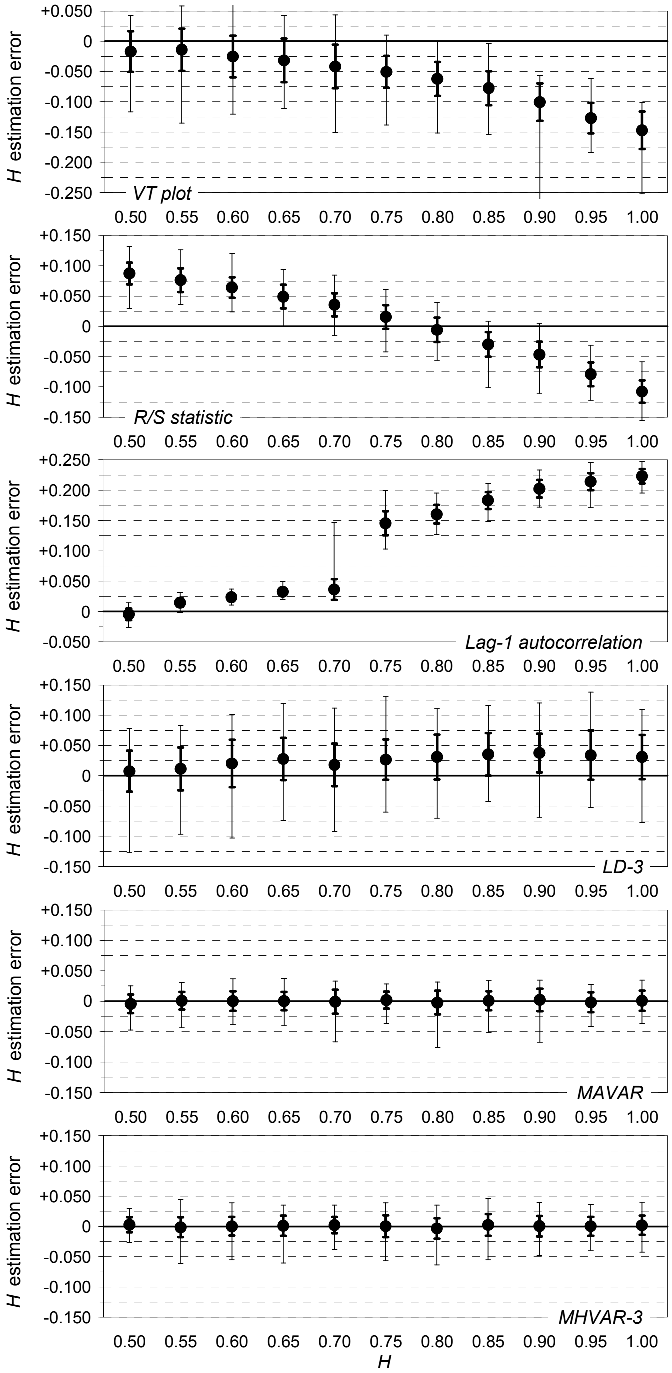

Fig. 2: Absolute estimation error of H attained by six methods (N=1024, mean, standard deviation and min-max excursion out of 100 samples).

are mostly affected by higher uncertainty. Estimates by lag-1 autocorrelation, finally, suffer a very heavy bias (up to 0.25 in estimating H values close to 1) and are by far the least accurate. Some more specific comment may be also made.

- *Long sequences* (N=131072). The mean of $\sigma_{\Delta i}$ of MHVAR-3 estimates is -22% than that of MAVAR, which in turn is -14% than that of LD-3. This confidence gain is significant, as it is computed over 1100 independent estimates. The mean of $\sigma_{\Delta i}$ of VT plot and R/S statistic estimates is four times wider than that of LD-3, MAVAR and MHVAR-3.
- *Short sequences* (N = 1024). The mean of $\sigma_{\Delta i}$ of MHVAR-3 estimates is -3% than that of MAVAR, which is -54% than that of LD-3. This last method, on short sequences, gives the estimates affected by highest uncertainty.

V. Conclusions

In this paper, by evaluation on thousands of pseudo-random LRD data series generated with assigned values of H, we compared the H estimation accuracy attained by some of the most widely used methods for LRD traffic analysis: viz. VT

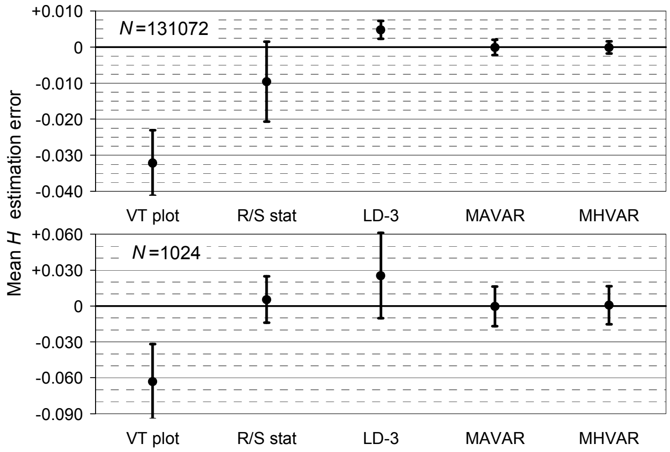

Fig. 3: Average mean E[$m_{\Delta i}$] and standard deviation E[$\sigma_{\Delta i}$] (i = 0, …, 10) of the H estimation errors attained by five methods (R(1) out of scale, average results on 100 pseudo-random sequences for each of 11 values H_i).

plot, R/S statistic, lag-1 autocorrelation, LD, MAVAR and MHVAR-3. These compared accuracy figures may be valuable to researchers involved in traffic measurement and characterization or, more generally, in fractional noise estimation.

Extensive simulations showed that MAVAR and MHVAR-3 achieve the best confidence and are not biased in H estimation. On long sequences (N = 131072), the mean standard deviation of 1100 MHVAR-3 estimates resulted 22% smaller than that of MAVAR, which in turn was 14% smaller than that of LD-3. On short sequences (N = 1024), MHVAR-3 and MAVAR attained similar confidence, far better than LD-3 (mean deviation 54% smaller). This superior performance is even more significant, if we also consider the LD-3 estimation bias.

Estimates by VT plot and R/S statistic resulted affected by significant bias and higher uncertainty: for N = 131072, standard deviation of both resulted four times wider than that of LD-3, MAVAR and MHVAR-3. Estimates by lag-1 autocorrelation, finally, exhibited strong bias (up to 0.25 in estimating H values close to 1) and were by far the least accurate.

The empirical accuracy figures presented in this paper further extend results provided in [24]. MAVAR and MHVAR-3 are the most accurate estimators of LRD parameters H and γ, in terms of both confidence and bias, among all methods considered in this study. Their computational complexity is comparable to that of other methods, in particular LD, since estimators (13)(14) can be computed recursively [12].

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