Optical and Transport Networks

Prof. Stefano Bregni **II Exam 2025-26 – 4 February 2026**

Last and first name: _(capital letters)_

(signature)

Matriculation number:

NB: In any exercise, any answer not justified adequately, even with few words, will not be considered.

Problem 1
(Solve on this sheet in the space provided) (6 points)

Consider a Passive Optical Network reaching up to N users at variable distances from the Optical Line Termination (OLT) via a variable number of splitters, with an asymmetric tree topology according to the scheme in figure.

The line from the OLT is cross-connected via an Optical Distribution Frame (ODF) to the PON. An Optical Amplifier (OA), if needed, may be added after the ODF at the Point-of-Presence (PoP). After a first single feeder fibre segment with length l_1, another ODF (Fibre Flexibility Point, FFP) cross-connects to the PON. The fibre segments between the FFP and the following splitters have length l_2, l_3, l_4, l_5, respectively. The length of other segments of fibres connecting network elements is negligible. The Optical Network Terminations (ONT) can be connected at the output of any splitter at the three stages (A, B, C).

Assume the following data for the PON elements:
- fibre with attenuation $\alpha = 0.5$ dB/km;
- $l_1 = 2$ km, $l_2 = 2$ km, $l_3 = 2$ km, $l_4 = 10$ km, $l_5 = 10$ km;
- OLT transmission power $P_{TX} = 100$ µW;
- splitter insertion loss $\alpha_s = 1$ dB;
- power loss by each couple of optical connectors $\alpha_c = 0.5$ dB (connections marked with dots in figure);
- sensitivity of ONT receivers $P_{RX} > -33$ dBm, with at least 6 dB of safety margin to be guaranteed;
- optional OA gain $+A$ [dB] (excluding the additional attenuation $2\alpha_c$ introduced by its two couples of connectors);
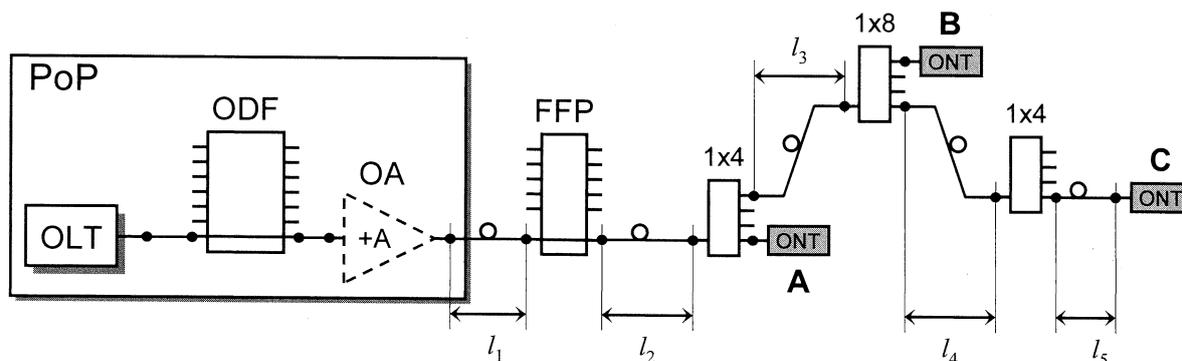
a) Evaluate the maximum _Differential Path Loss_ [dB] between ONTs.

b) Evaluate the power P_{RX} [W] received by the farthest ONT in position C without OA.

c) Determine if it is necessary to add an OA, to make the power P_{RX} received by the farthest ONT not less than the minimum power required at the ONT receiver.
   - If the OA is necessary, calculate the minimum OA gain (excluding the additional attenuation $2\alpha_c$ introduced by its two couples of connectors) required.
   - Otherwise, if the system is feasible without OA, calculate the maximum length L of the last fiber segment, to have P_{RX} at any ONT not less than the sensitivity of receivers including the safety margin.

d) The OLT transmits a timing signal downstream to synchronize all ONTs, which are equipped with PLL-based slave clocks. Assume that the fiber has refractive index $n = 1.5$ and coefficient of variation of the refractive index vs. temperature $\partial n / \partial \theta = +5 \cdot 10^{-6}$/K. What is the maximum _Time Alignment Error_ (TAE) between two ONTs at stage C synchronized by the received signals, due to refractive index variation, assuming one-way synchronization OLT→ONT, in case the fibre paths connecting the OLT to the ONTs may reach temperature difference up to $\Delta\theta = 20°C$ between them, due to presence or absence of thermal isolation?

a).
$$OPL = P_{Rx/A} - P_{Rx/c} = \quad [dB]$$

$$= \alpha(l_3 + l_4 + l_5) + 5\alpha_c + (\alpha_5 + 5) + (\alpha_5 + 6) = 30.5 dB$$

b) $P_{Rx/c} = P_{Tx} - 12\alpha_c - (\alpha_5 + 5) - 2(\alpha_5 + 6) - \alpha \sum_1^5 l_i = \quad [dB]$

$$= \quad -53 \, dBm \simeq 5 \, nW \qquad P_{Tx} = -10 \, dBm$$

c) $P_{Rx/c} < -27 \, dBm \Rightarrow OA$

$$A \geqslant 26 \, dB + 2\alpha_c = 27 \, dB \; (!)$$

d) $\quad \tau = \dfrac{1}{v} = L\dfrac{M}{c} \to \Delta\tau = L\dfrac{\Delta M}{c} = L\dfrac{\partial M}{\partial\theta}\Delta\theta\dfrac{1}{c} = 7.3 \, ms$

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Problem 2
(Solve on this sheet in the space provided) (5 points)

Consider the frame alignment algorithm represented by the diagram below (A_0 state: system aligned in service; B state: alignment lost). The frame aligner operates on a test framed signal with nominal frequency $f_0 = 139.264$ Mbit/s, frame length $L_m = 2928$ bit, random content everywhere in all frames except the alignment word (12 bits during both hunting and maintenance). The test signal is affected by random transmission errors, uncorrelated and with rate $\varepsilon = 10^{-5}$.

a) Let the system be aligned and in service (A_0 state). What are the limit values of the frame alignment parameters α and δ, in order to have probability of *forced loss of alignment* $P_{FL} < 10^{-20}$?
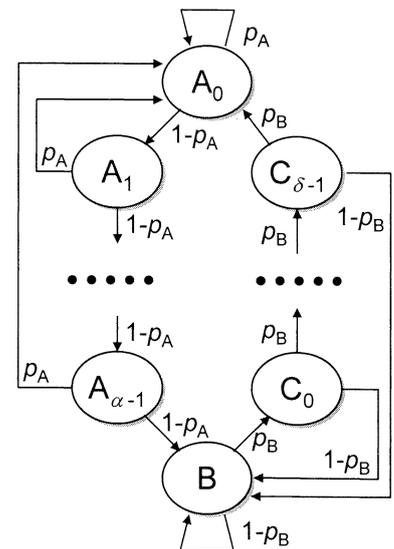
$$P_A = (1-\varepsilon)^{12} \cong 1 - 12\varepsilon$$

$$P_{FL} = (1-P_A)^\alpha < 10^{-20}$$

$$(12\varepsilon)^\alpha < 10^{-20} \qquad \alpha \log_{10}(12\varepsilon) < -20$$

$$\Rightarrow \alpha \geq 6$$

b) Let the system be out of alignment (B state). What are the limit values of the frame alignment parameters α and δ, in order to have the probability of *fake alignment* $P_{FA} < 10^{-20}$?

$$P_B = \frac{1}{2^{12}}$$

$$P_{FA} = P_B^{\delta+1} < 10^{-20}$$

$$(\delta+1) \log_{10} \frac{1}{2^{12}} < -20 \qquad \Rightarrow \delta \geq 5$$

Problem 3
*(Solve on this sheet in the space provided)* *(8 points)*

a) Let $s(t)$ be a pseudo-sinusoidal timing signal with nominal frequency $v_0 = 0.4$ Hz and *instantaneous frequency error* $\Delta v(t) = v(t) - v_0$ as plotted in figure.

Where possible, plot on the graphs at right:

- the timing signal $s(t)$;
- the *Total Phase* $\Phi(t)$ of $s(t)$ and of the ideal timing signal with frequency v_0, both starting from $\Phi(0) = 0$;
- the *Time Error* TE(t) with respect to an ideal timing signal with frequency v_0, starting from TE(0)=0, with the convention that positive TE denotes time advance.
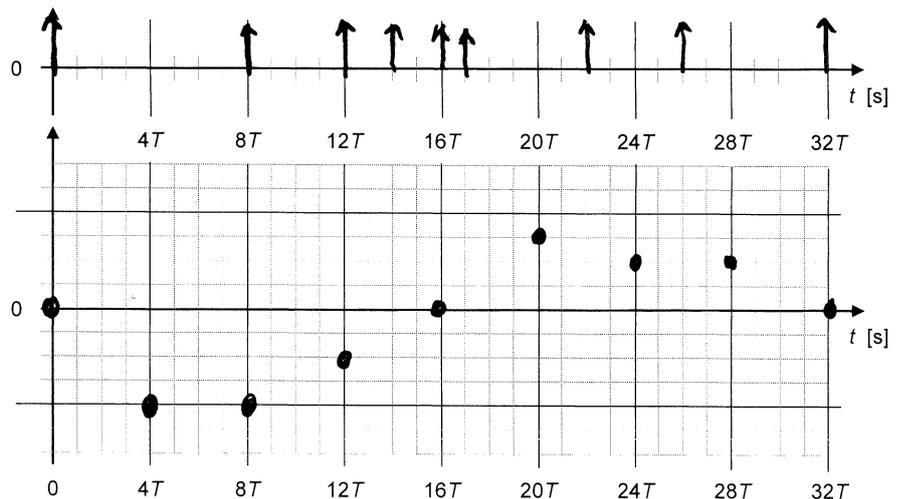
$$v(t) = ? \frac{y \, d\phi}{3\Omega \, r_{\ell}} = 0.233 \text{ Hz}$$

$$T_0 = ?.5 \text{ sec}$$
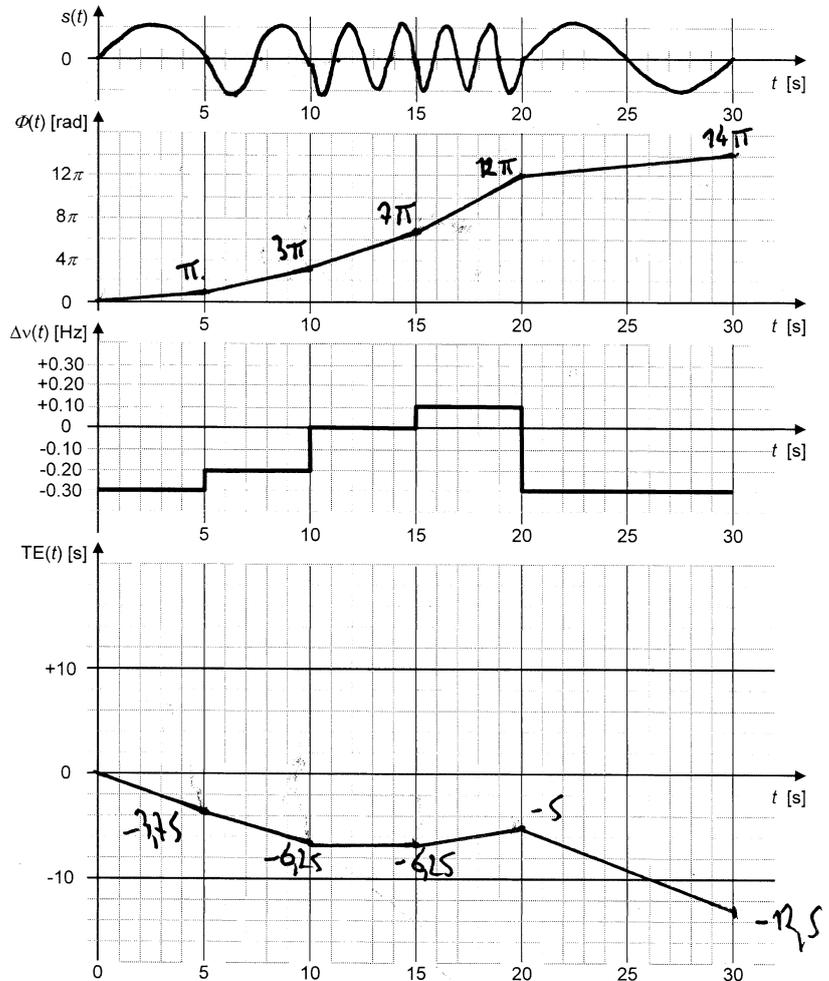
b) A source transmits packets to a destination with constant rate every $4T$. Packets are supposed short enough to have duration negligible compared to T. Nine packets numbered $k = 0, 1, ..., 8$ are transported over the network and arrive to their destination with the sequence of inter-arrival times $\{y_k\} = (8T, 4T, 2T, 2T, T, 5T, 4T, 6T)$, where y_k is the inter-arrival time between packet k and the next one.

Plot on the graph the PDV values $e[k]$, measured in T units, at the instants $t_k = k(4T)$ of ideal arrival of packets, besides the latency of packet 0, starting from the initial point $e[0] = 0$, with the convention that positive PDV denotes time advance.

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Problem 4
*(Solve on this sheet in the space provided) (17 points)*

1) Consider a Slave Clock based on a *second-order PLL* system, as shown in the figure below at left. Let us denote as $s_{IN}(t)$ and $s_{OUT}(t)$ its input and output timing signals, respectively, and as $\phi_{IN}(t)$ and $\phi_{OUT}(t)$ their respective *phase errors* vs. the Total Phase $\Phi(t)$ of the ideal timing signal $s(t) = A \sin 2\pi\nu_0 t$ considered as common reference in this model, having frequency ν_0. Therefore: $s_{IN}(t) = A \sin (2\pi\nu_0 t + \phi_{IN}(t))$ and $s_{OUT}(t) = A \sin (2\pi\nu_0 t + \phi_{OUT}(t))$. The closed-loop transfer function of the PLL is the standard $H(s)$ plotted below at right ($\omega_n = 20\pi\,\text{s}^{-1}$, $\zeta = 0.1$). *(4 points)*
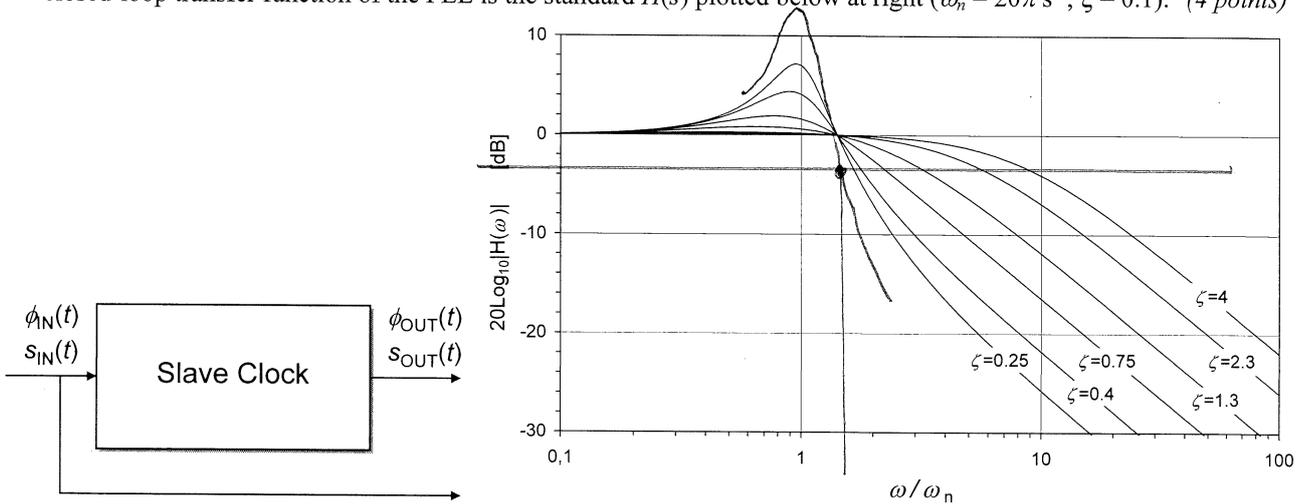
- What is the cutoff frequency (-3 dB point) [Hz] of the slave clock?

$f_n = 10\,\text{Hz} \qquad f_c \approx 15\,\text{Hz}$

- The input timing signal $s_{IN}(t)$ exhibits a phase error $\phi_{IN}(t)$ vs. the Total Phase of the ideal timing signal $s(t)$ as shown in the graphs below (sinusoidal). Plot on the same graphs the output phase $\phi_{OUT}(t)$ ignoring the initial phase alignment between input and output. Explain your responses and express your considerations.
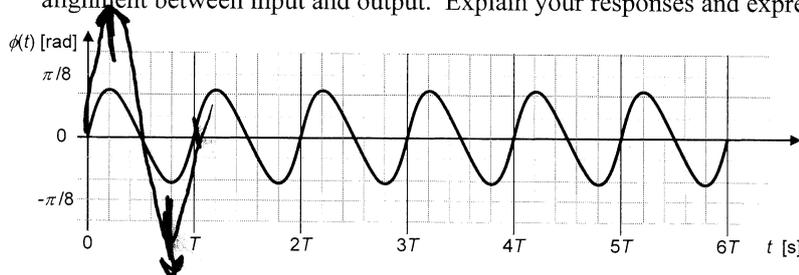
T = 0.1 s

$> 10\,\text{dB}\,\text{gain}$
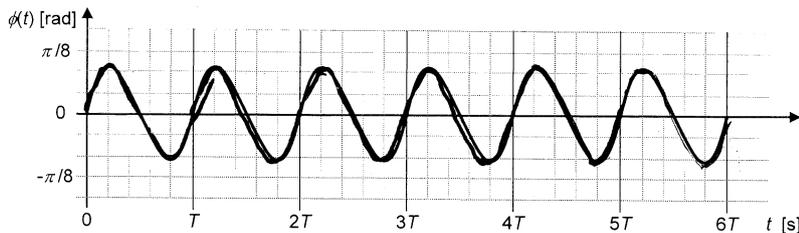
T = 1000 s

2) What is the best efficiency (payload utilization factor) achievable to transport a user data signal with rate 1.25 Gbit/s by way of VC-4 VCAT in SDH systems? (VC-4 frame: 261×9. STM-1 rate: 155.520 Mbit/s) *(2 points)*

VC-4 net capacity: $155.520 \text{ Mb/s} \dfrac{260}{270} = 149.760 \text{ Mb/s } (C_{c4})$

VC-4-9v capacity: $C_{c4} \cdot 9 = 1347.84 \text{ Mb/s} \Rightarrow \eta = 92.7 \%$

3) What is the *interleaving* in the BIP(n, m) code? What is its purpose? What is a reasonable criterion to determine n and m? *(3 points)*

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4) In planning an FTTH network, the advantage of PON vs. P2P is usually claimed to consist mainly in the lower total cost of infrastructure deployment. Discuss why the difference of cost is not simply proportional to the difference of total fiber length. What is the impact on the Point-of-Presence in the Central Office?   *(2 points)*

5) Why the scrambler to pseudo-randomize user data before mapping in transport frames (e.g., IP over SDH or OTN) is *self-synchronizing*, but the scrambler to pseudo-randomize such the SDH/OTN signal before transmitting over an optical or radio system is *additive*? Discuss pros and cons of the self-synchronizing vs. additive alternative schemes. *(3 points)*

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6) Describe the different fields of applications of NTP vs. PTP for time synchronization. What protocol achieves best accuracy? In any condition, or depending on the environment where it is applied? *(3 points)*