

Optical and Transport Networks

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I Exam 2025-26 – 22 January 2026

Last and first name:

(capital letters)

(signature)

Matriculation number:

NB: In any exercise, any answer not justified adequately, even with few words, will not be considered.

Problem 1

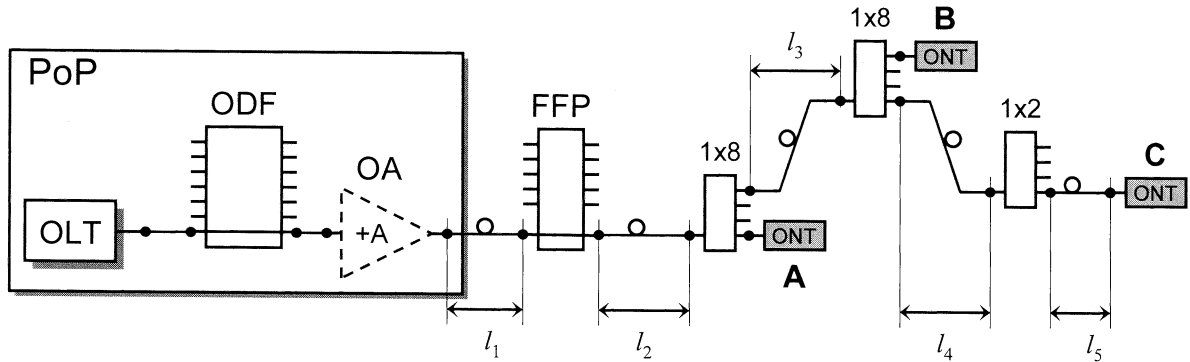
(Solve on this sheet in the space provided) (6 points)

Consider a Passive Optical Network reaching up to N users at variable distances from the Optical Line Termination (OLT) via a variable number of splitters, with an asymmetric tree topology according to the scheme in figure.

The line from the OLT is cross-connected via an Optical Distribution Frame (ODF) to the PON. An Optical Amplifier (OA), if needed, may be added after the ODF at the Point-of-Presence (PoP). After a first single feeder fibre segment with length l_1 , another ODF (Fibre Flexibility Point, FFP) cross-connects to the PON. The fibre segments between the FFP and the following splitters have length l_2, l_3, l_4, l_5 , respectively. The length of other segments of fibres connecting network elements is negligible. The Optical Network Terminations (ONT) can be connected at the output of any splitter at the three stages (A, B, C).

Assume the following data for the PON elements:

- fibre with attenuation $\alpha = 0.25$ dB/km;
- $l_1 = 10$ km, $l_2 = 2$ km, $l_3 = 3$ km, $l_4 = 3$ km, $l_5 = 5$ km;
- OLT transmission power $P_{TX} = 3$ mW;
- splitter insertion loss $\alpha_s = 1$ dB;
- power loss by each couple of optical connectors $\alpha_c = 0.5$ dB (connections marked with dots in figure);
- sensitivity of ONT receivers $P_{RX} > -33$ dBm, with at least 6 dB of safety margin to be guaranteed;
- optional OA gain $+A$ [dB] (excluding the additional attenuation $2\alpha_c$ introduced by its two couples of connectors);



- Evaluate the maximum *Differential Path Loss* [dB] between ONTs.
- Evaluate the power P_{RX} [W] received by the farthest ONT in position C without OA.
- Determine if it is necessary to add an OA, to make the power P_{RX} received by the farthest ONT not less than the minimum power required at the ONT receiver.
 - If the OA is necessary, calculate the minimum OA gain (excluding the additional attenuation $2\alpha_c$ introduced by its two couples of connectors) required.
 - Otherwise, if the system is feasible without OA, calculate the maximum length L of the last fiber segment, to have P_{RX} at any ONT not less than the sensitivity of receivers including the safety margin.
- The OLT transmits a timing signal downstream to synchronize all ONTs, which are equipped with PLL-based slave clocks. Assume that the fiber has refractive index $n = 1.6$ and coefficient of fractional variation of length vs. temperature $\frac{1}{l} \frac{\partial l}{\partial \theta} = +0.8 \cdot 10^{-6}/K$. What is the maximum *Time Alignment Error* (TAE) between two ONTs at stage C

synchronized by the received signals, due to fiber length variation, assuming one-way synchronization OLT→ONT, in case the fibre paths connecting the OLT to the ONTs may reach temperature difference up to $\Delta\theta = 20^\circ\text{C}$ between them, due to presence or absence of thermal isolation?

$$a) \max \Delta PL = P_{Rx/A} - P_{Rx/C} [\text{dB}]$$

$$= \alpha(l_3 + l_4 + l_5) + 5\alpha_c + (\alpha_s + 9) + (\alpha_s + 3) = 19.25 \text{ dB}$$

$$b) P_{Rx/C} = P_{Tx} - 12\alpha_c - 2(\alpha_s + 9) - (\alpha_s + 3) - \alpha(l_1 + l_2 + l_3 + l_4 + l_5) = [\text{dB}]$$

$$= -30.98 \text{ dBm} = 0.8 \mu\text{W}$$

$$P_{Tx} = 4.77 \text{ dBm}$$

$$c) P_{Rx/C} < -27 \text{ dBm} \Rightarrow \text{OA}$$

$$A > \sim 4 \text{ dB} + 2\alpha_c = 5 \text{ dB}$$

$$d) \tau = \frac{L}{v} = L \frac{n}{c} \rightarrow \Delta\tau = \Delta L \frac{n}{c} \quad L = l_3 + l_4 + l_5$$

$$\Delta L = L \left(\frac{1}{L} \frac{\partial L}{\partial \theta} \right) \Delta\theta =$$

$$= 0.176 \text{ m}$$

$$\Delta\tau \approx \sim 0.94 \text{ ns}$$

Problem 2

(Solve on this sheet in the space provided) (5 points)

Let $s(t)$ be a non-ideal timing signal generated by a clock with initial instantaneous frequency set to the nominal frequency $\nu(0) = \nu_0 = 1$ Hz and coefficient of quadratic frequency drift $D = 10^{-10}/(\text{month})^2$, that is $\nu(t) = \nu_0 + Dt^2\nu_0$

a) Derive the analytical expression of the *Total Phase* $\Phi(t)$ (where t [days]) knowing that $\Phi(0) = 0$.

$$\Phi(t) = 2\pi\nu_0 \int_0^t (1 + Dt^2) dt = 2\pi\nu_0 \left(t + \frac{D}{3}t^3 \right)$$

b) Under the assumption that the frequency drift remains quadratic with coefficient D indefinitely, evaluate the *Time Error* $TE(t)$ [s] measured by this clock at $t = 1$ year (12 months in year, 30 days in a month) with respect to an ideal timing signal with constant frequency ν_0 and same phase at $t = 0$.

$$T(t) = \frac{\Phi(t)}{2\pi\nu_0} = t + \frac{D}{3}t^3$$

$$TE(t) = T(t) - t = \frac{D}{3}t^3$$

For $t = 12$ months:

$$\begin{aligned} TE(12 \text{ mo}) &= \frac{1}{3} 10^{-10} / \text{month}^2 \cdot (12 \text{ months})^3 \\ &= 0.15 \text{ sec} \end{aligned}$$

Problem 3

(Solve on this sheet in the space provided) (8 points)

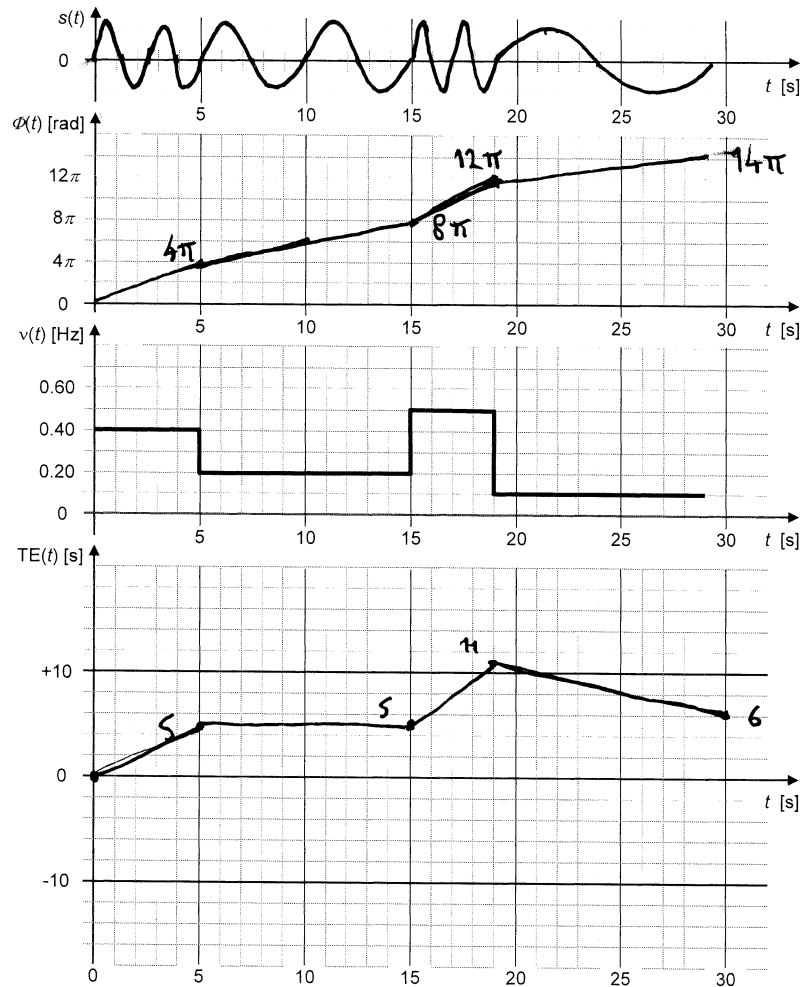
- a) Let $s(t)$ be a pseudo-sinusoidal timing signal with nominal frequency $\nu_0 = 0.2$ Hz and *instantaneous frequency* $\nu(t)$ as plotted in figure.

Where possible, plot on the graphs at right:

- the timing signal $s(t)$;
- the *Total Phase* $\Phi(t)$ of $s(t)$ and of the ideal timing signal with frequency ν_0 , both starting from $\Phi(0) = 0$;
- the *Time Error* $TE(t)$ with respect to an ideal timing signal with frequency ν_0 , starting from $TE(0) = 0$, with the convention that positive TE denotes time advance.

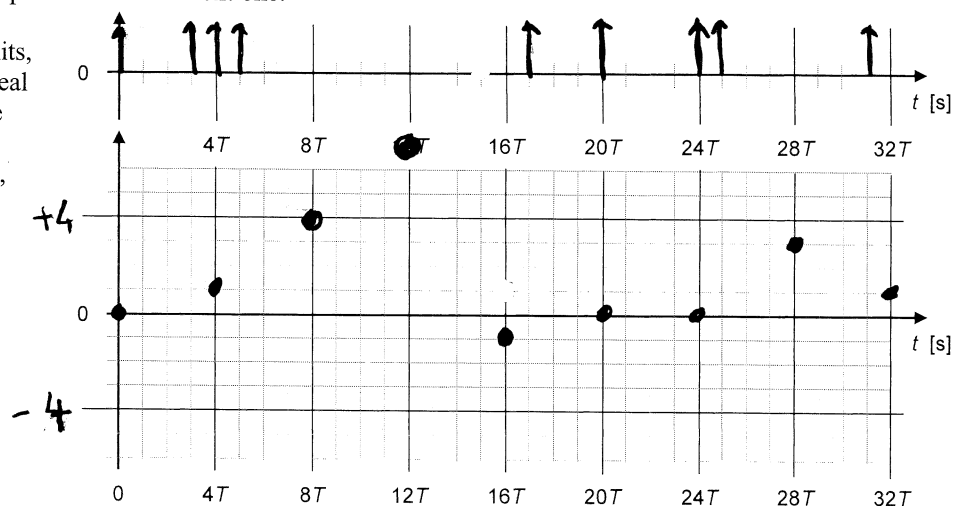
$$\bar{\nu}(t) = \frac{7 \text{ cycles}}{29 \text{ sec}} = 0.241 \text{ Hz}$$

$$T_0 = 5 \text{ sec}$$



- b) A source transmits packets to a destination with constant rate every $4T$. Packets are supposed short enough to have duration negligible compared to T . Nine packets numbered $k = 0, 1, \dots, 8$ are transported over the network and arrive to their destination with the sequence of inter-arrival times $\{y_k\} = (3T, T, T, 12T, 3T, 4T, T, 6T)$, where y_k is the inter-arrival time between packet k and the next one.

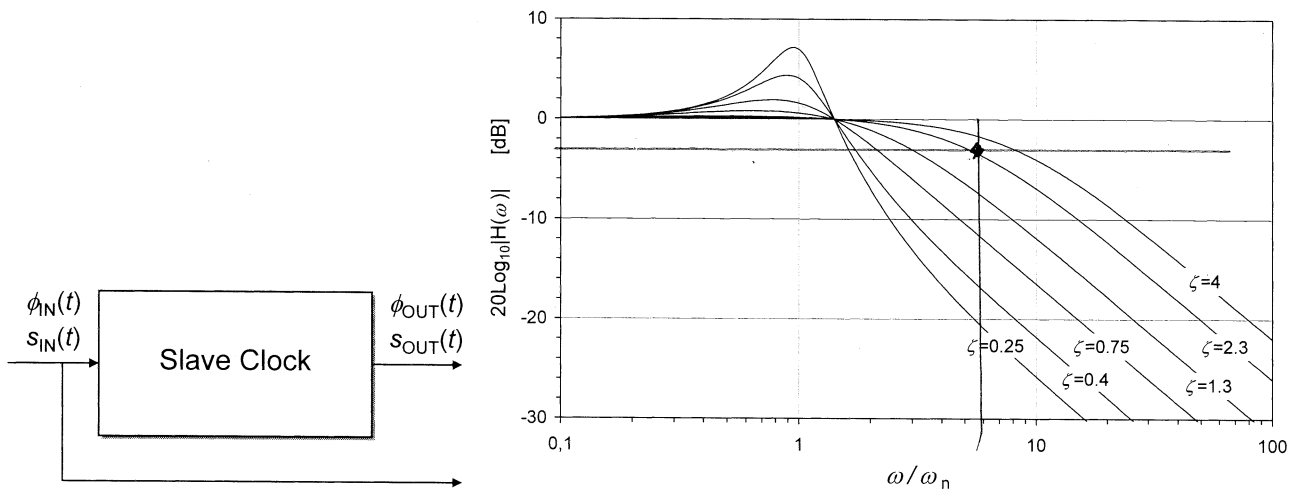
Plot on the graph the PDV values $e[k]$, measured in T units, at the instants $t_k = k(4T)$ of ideal arrival of packets, besides the latency of packet 0, starting from the initial point $e[0] = 0$, with the convention that positive PDV denotes time advance.



Problem 4

(Solve on this sheet in the space provided) (17 points)

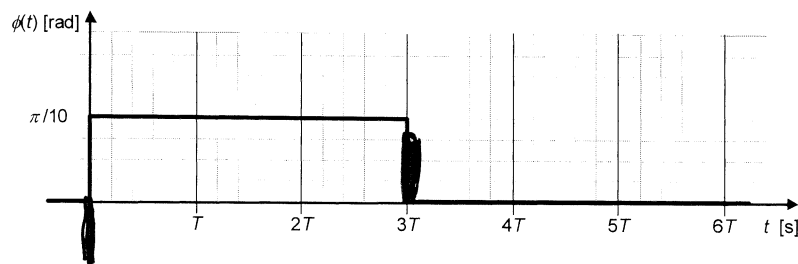
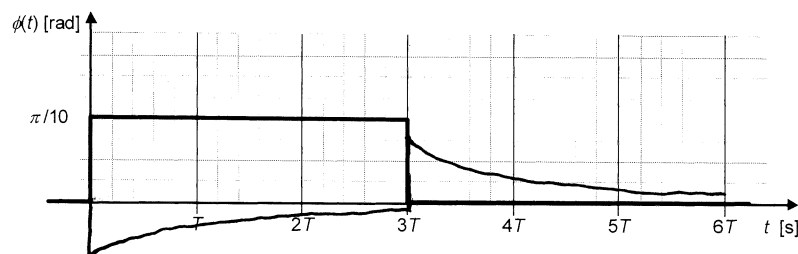
- 1) Consider a Slave Clock based on a *second-order PLL* system, as shown in the figure below at left. Let us denote as $s_{IN}(t)$ and $s_{OUT}(t)$ its input and output timing signals, respectively, and as $\phi_{IN}(t)$ and $\phi_{OUT}(t)$ their respective *phase errors* vs. the Total Phase $\Phi(t)$ of the ideal timing signal $s(t) = A \sin 2\pi\nu_0 t$ considered as common reference in this model, having frequency ν_0 . Therefore: $s_{IN}(t) = A \sin (2\pi\nu_0 t + \phi_{IN}(t))$ and $s_{OUT}(t) = A \sin (2\pi\nu_0 t + \phi_{OUT}(t))$. The closed-loop transfer function of the PLL is the standard $H(s)$ plotted below at right ($\omega_n = 3300\pi \text{ s}^{-1}$, $\zeta = 2.3$). (4 points)



- What is the cutoff frequency (-3 dB point) [Hz] of the slave clock?

$\sim 10 \text{ kHz}$

- The input timing signal $s_{IN}(t)$ exhibits a phase error $\phi_{IN}(t)$ vs. the Total Phase of the ideal timing signal $s(t)$ as shown in the graphs below. Plot on the same graph the phase error $\phi_{OUT}(t) - \phi_{IN}(t)$. Explain your responses and express your considerations.



- 2) Before mapping the client digital signal into the frames of the transport network, user data are pseudo-randomized with a self-synchronizing scrambler (public). A malicious user wishes to enforce a particular data pattern carried over the transport systems, knowing the scrambler scheme and order, but not how memory cells are initialized. What should be the scrambler order, in order to make the probability that the malicious user matches the desired pattern at the first attempt $P < 10^{-15}$? (2 points)

$$\frac{1}{2^n} < 10^{-15}$$

$$n > 15 \log_2 10$$

$$n \geq 50$$

- 3) Define the efficiency η of bit error rate estimation by a BIP(n, m) code. Explain why it is a decreasing function of the line bit error rate ε , from $\eta \rightarrow 1$ (for $\varepsilon \rightarrow 0$) to $\eta \rightarrow 0$. Why consecutive errors may affect negatively the efficiency of bit error rate estimation? (2 points)

- 4) Define what are *forced loss of alignment*, *real loss of alignment*, *fake alignment*. Explain when they occur and how to avoid them. (3 points)

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- 5) Describe an experimental set-up and procedure to measure the *Time Error* between the input and output timing signals $s_{\text{IN}}(t)$ and $s_{\text{OUT}}(t)$ of the slave clock in Question 4.1). How does the shape of the two timing signals (square or sinusoidal) affect the measurement? *(3 points)*

- 6) Describe the *power-law model* of phase and frequency instabilities in precision oscillators. (3 points)