Optical and Transport Networks

Prof. Stefano Bregni **II Exam 2024-25 – 13 February 2025**

Last and first name: *(capital letters)*
(signature)

Matriculation number:

NB: In any exercise, any answer not justified adequately, even with few words, will not be considered.

Problem 1

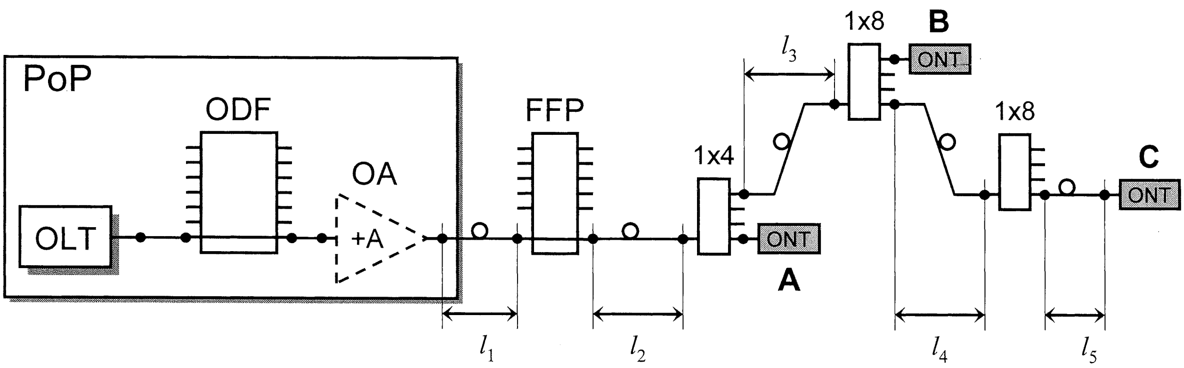
(Solve on this sheet in the space provided) (7 points)

Consider a Passive Optical Network reaching up to N users at variable distances from the Optical Line Termination (OLT) via a variable number of 1×4 and 1×8 splitters, with an asymmetric tree topology according to the scheme in figure.

The line from the OLT is cross-connected via an Optical Distribution Frame (ODF) to the PON. An Optical Amplifier (OA), if needed, may be added after the ODF at the Point-of-Presence (PoP). After a first single feeder fibre segment with length l_1, another ODF (Fibre Flexibility Point, FFP) cross-connects to the PON. The fibre segments between the FFP and the following splitters have length l_2, l_3, l_4, l_5, respectively. The length of other segments of fibres connecting network elements is negligible. The Optical Network Terminations (ONT) can be connected at the output of any splitter at the three stages (A, B, C).

Assume the following data for the PON elements:

- fibre with attenuation $\alpha = 0.4$ dB/km;
- $l_1 = 2$ km, $l_2 = 8$ km, $l_3 = 8$ km, $l_4 = 8$ km, $l_5 = 10$ km;
- OLT transmission power $P_{TX} = 2$ mW;
- splitter insertion loss $\alpha_s = 1$ dB;
- power loss by each couple of optical connectors $\alpha_c = 0.5$ dB (connections marked with dots in figure);
- sensitivity of ONT receivers $P_{RX} > -33$ dBm, with at least 6 dB of safety margin to be guaranteed;
- optional OA gain $+A$ [dB] (excluding the additional attenuation $2\alpha_c$ introduced by its two couples of connectors);



a) Evaluate the maximum *Differential Path Loss* [dB] between ONTs.

b) Evaluate the power P_{RX} [W] received by the farthest ONT in position C without OA.

c) Determine if it is necessary to add an OA, to make the power P_{RX} received by the farthest ONT not less than the minimum power required at the ONT receiver.
   - If the OA is necessary, calculate the minimum OA gain (excluding the additional attenuation $2\alpha_c$ introduced by its two couples of connectors) required.
   - Otherwise, if the system is feasible without OA, calculate the maximum length L of the last fiber segment, to have P_{RX} at any ONT not less than the sensitivity of receivers including the safety margin.

d) Consider the case a 2.5 Gbit/s GPON signal is transmitted over the fibres by the OLT to the ONTs. Assume that the coefficient of variation of the refractive index vs. temperature of the fibre is $\partial n/\partial\theta = +5\cdot10^{-6}$/K. What is the maximum *Time Alignment Error* (TAE) between two ONTs at stage C synchronized by the received signals, assuming one-way synchronization OLT→ONT, in case the fibre paths connecting the OLT to the two ONTs may reach temperature difference up to $\Delta\theta = 20$°C between them, due to presence or absence of thermal isolation?

a) $\max DPL = P_{Rx/A} - P_{Rx/C}$ [dB]

$= \alpha(l_3 + l_4 + l_5) + 5\alpha_c + 2(\alpha_s + 9) = 32,9$ dB

b) $P_{Rx/C} = P_{TX} - 12\alpha_c - (\alpha_s + 6) - 2(\alpha_s + 9) - \alpha \sum_1^5 l_i$ [dB]

$= -44,4$ dBm $= 36,3$ nW $\qquad P_{TX} = 3$ dBm

c) $P_{Rx/C} < -27$ dBm $\Rightarrow$ $\circ A$

$A \geq 17.4$ dB $+ 2\alpha_c = 18.4$ dB

d) $\tau = \frac{L}{V} = L\frac{n}{c} \rightarrow \Delta\tau = \frac{L}{c}\Delta n \qquad L = \sum_1^5 l_i = 36$ Km

$\Delta n = \left(\frac{+5 \cdot 10^{-6}}{K}\right) \cdot (20\ °C)$

$= 10^{-4}$

$\Delta\tau = 12$ ns

Last and first name: *(capital letters)*
(signature)

Matriculation number:

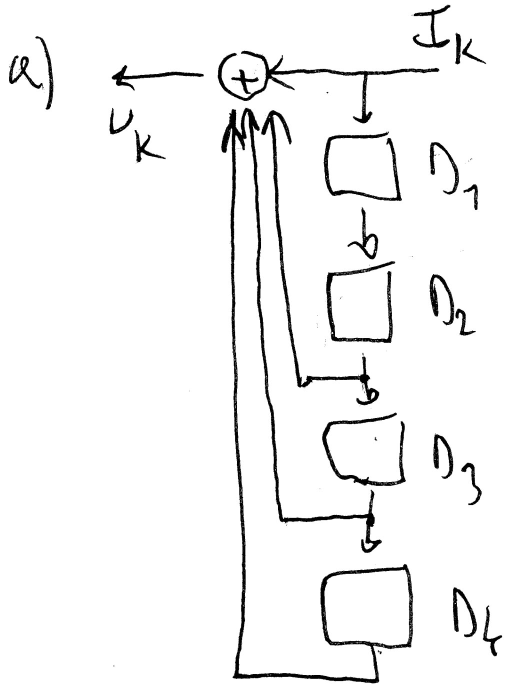
Problem 2

(Solve on this sheet in the space provided) (5 points)

a) Draw the scheme of a *self-synchronizing descrambler* with characteristic polynomial $P(x) = x^4 + x^3 + x^2 + 1$.

b) Initialize the delay cells D_i (i = 1, 2, 3, 4) as {1, 1, 1, 1} at the initial step k = 0. Feed it with the bit sequence to descramble $\{I_k\}$ = {1, 1, 1, 1, 0, 0, 1, 0, 0, 0, 0, 1, 1, 0, …} (periodic with period Π = 14). Calculate the descrambled bit sequence $\{U_k\}$.

a)



k	I_k	D_{1k}	D_{2k}	D_{3k}	D_{4k}	U_k
0	1	1	1	1	1	0
1	1	1	1	1	1	0
2	1	1	1	1	1	0
3	1	1	1	1	1	0
4	0	1	1	1	1	1
5	0	0	1	1	1	1
6	1	0	0	1	1	1
7	0	1	0	0	1	1
8	0	0	1	0	0	1
9	0	0	0	1	0	1
10	0	0	0	0	1	1
11	1	0	0	0	0	1
12	1					
13	0					
14	1					

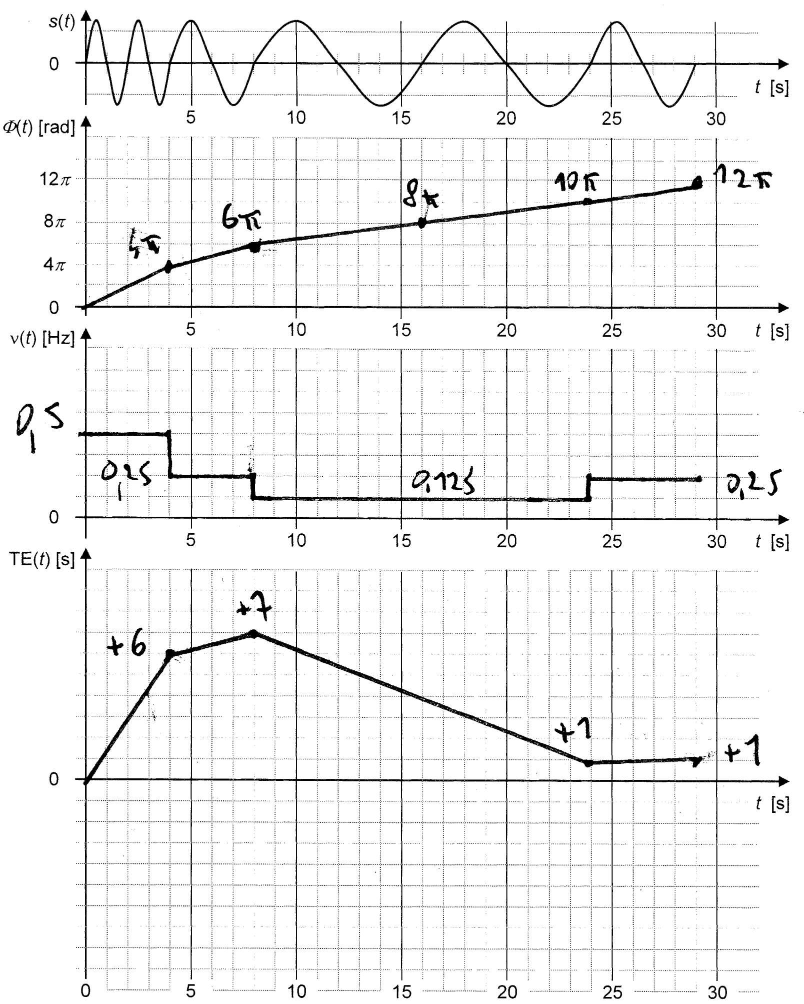
Problem 3

(Solve on this sheet in the space provided) (7 points)

a) Let $s(t)$ be a pseudo-sinusoidal timing signal as plotted in figure, having nominal frequency $\nu_0 = 0.2$ Hz.

- Calculate the average frequency of $s(t)$ over the interval $0 \le t \le 29$ s.

Where possible, plot on the graphs at right:
- the *Total Phase* $\Phi(t)$;
- the *instantaneous frequency* $\nu(t)$;
- the *Time Error* TE(t) = T(t)-t, starting from TE(0)=0, with the convention that positive TE denotes time advance.

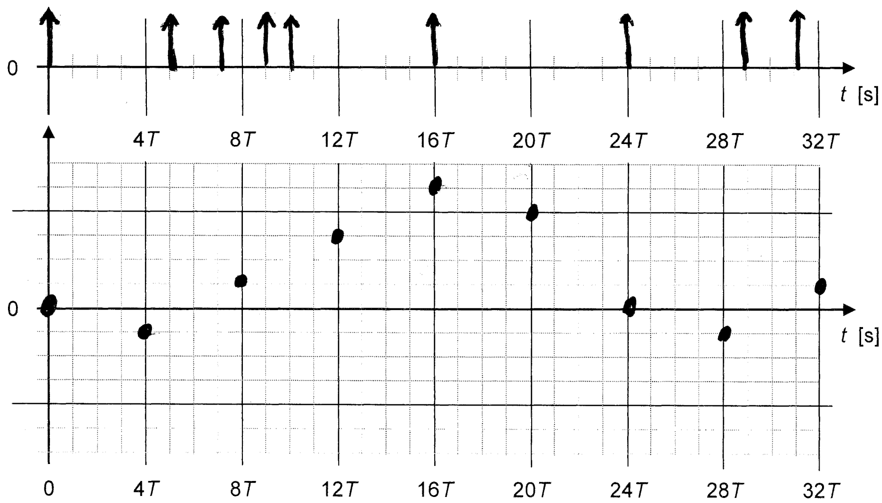


$\nu(t) = \frac{6 \text{ cycles}}{29 \text{ sec}} \simeq 0,2069$

$T_0 = 5$ sec

b) A source transmits packets to a destination with constant rate every $4T$. Packets are supposed short enough to have duration negligible compared to T. Nine packets numbered $k = 0, 1, \ldots, 8$ are transported over the network and arrive to their destination with the sequence of inter-arrival times $\{y_k\} = (5T, 2T, 2T, T, 6T, 8T, 5T, 2T)$, where y_k is the inter-arrival time between packet k and the next one.

Plot on the graph the PDV values $e[k]$, measured in T units, at the instants $t_k = k(4T)$ of ideal arrival of packets, besides the latency of packet 0, starting from the initial point $e[0] = 0$, with the convention that positive PDV denotes time advance.



Optical and Transport Networks

Prof. Stefano Bregni **II Exam 2024-25 – 13 February 2025**

Last and first name: *(capital letters)*

(signature)

Matriculation number:

Problem 4

(Solve on this sheet in the space provided) (17 points)

1) Consider the case an optical signal transmitted over a fibre of length L with refractive index $n = 1.5$. The coefficient of fractional variation of length vs. temperature of the fibre is $\frac{1}{l}\frac{\partial l}{\partial \theta} = +0.8 \cdot 10^{-6}$/K. The coefficient of variation of the refractive index vs. temperature of the fibre is $\partial n/\partial \theta = +5 \cdot 10^{-6}$/K. What is the ratio $\Delta\tau_n / \Delta\tau_L$ of the amplitudes of diurnal wander caused by the two effects if the temperature variation is $\Delta\theta = 10$°C? *(2 points)*

$\tau = \frac{1}{v} = L\frac{n}{c}$ $\quad \Delta\tau_n = L\frac{\Delta n}{c} = L\frac{\partial n}{\partial \theta} \cdot \Delta\theta \frac{1}{c} = L \cdot \frac{5 \cdot 10^{-5}}{3 \cdot 10^{5} \text{ km/s}} =$

$= L \cdot (1{,}67 \cdot 10^{-10} \text{ s/km})$

$\Delta\tau_L = \Delta L \frac{n}{c} = L\left(\frac{1}{l}\frac{\partial l}{\partial \theta}\right)\Delta\theta \frac{n}{c} = L \cdot \frac{8 \cdot 10^{-6} \cdot 1{,}5}{3 \cdot 10^{5} \text{ km/s}} =$

$= L \cdot (4 \cdot 10^{-11} \text{ s/km})$

$\frac{\Delta\tau_n}{\Delta\tau_L} = 4.17$

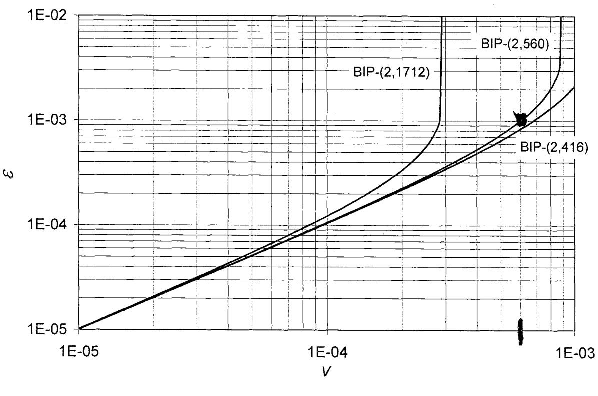
2) The figure below plots the calibration curves $\varepsilon(v)$ for adjusting BER estimates of bits 1, 2 of byte V5 of VC-11, VC-12 and VC-2, as provided respectively by codes BIP(2,416), BIP(2,560), BIP(2,1712). *(3 points)*

a) What is the maximum parity violation rate v, which can be detected by code BIP(2,416)?

b) If the bit error rate on the transmission line is BER = 10^{-3}, what is the approximate parity violation rate v detected by code BIP(2,560)? Explain why v is not equal to 10^{-3}.

a) $v \leq \frac{1}{2 \cdot 417} = 1.2 \cdot 10^{-3}$

b) $v = 7 \cdot 10^{-4} < 10^{-3}$



3) What is the *jitter* of a digital signal? Provide its definition and explain why it may cause errors on reception. *(2 points)*

4) For what reasons the PON solution is generally preferred over P2P Ethernet, in deploying an FTTX access system in a urban area? For better performance or for what implementation problems? *(2 points)*

Optical and Transport Networks

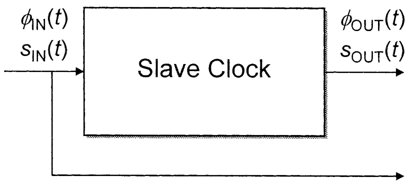
Prof. Stefano Bregni — **II Exam 2024-25 – 13 February 2025**

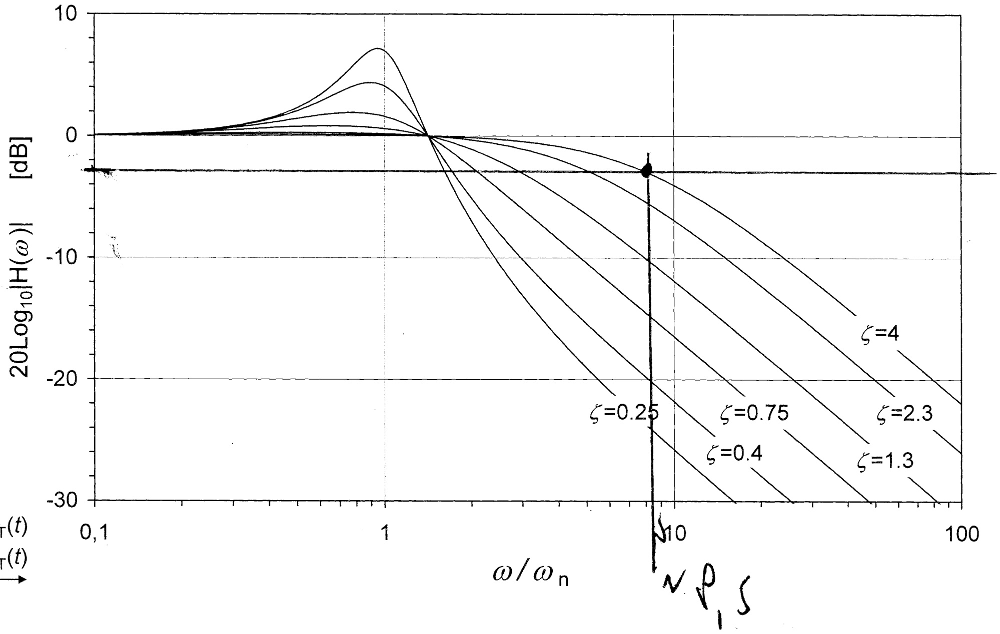
Last and first name: *(capital letters)*

(signature)

Matriculation number:

5) Consider a Slave Clock based on a *second-order PLL* system, as shown in the figure below at left. Let us denote as $s_{IN}(t)$ and $s_{OUT}(t)$ its input and output timing signals, respectively, and as $\phi_{IN}(t)$ and $\phi_{OUT}(t)$ their respective *phase errors* vs. the Total Phase $\Phi(t)$ of the ideal timing signal $s(t) = A \sin 2\pi\nu_0 t$ considered as common reference in this model, having frequency ν_0. Therefore: $s_{IN}(t) = A \sin (2\pi\nu_0 t + \phi_{IN}(t))$ and $s_{OUT}(t) = A \sin (2\pi\nu_0 t + \phi_{OUT}(t))$. The closed-loop transfer function of the PLL is the standard $H(s)$ plotted below at right ($\omega_n = 200\pi$ s^{-1}, $\zeta = 4$). *(4 points)*

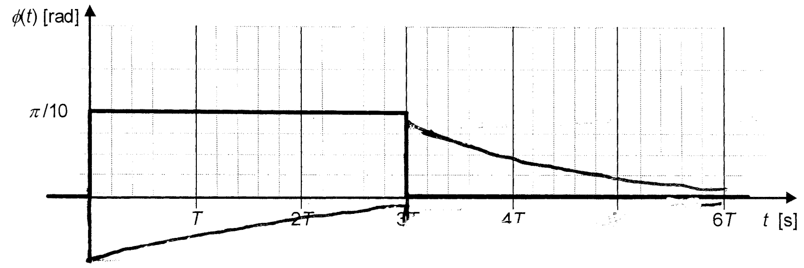




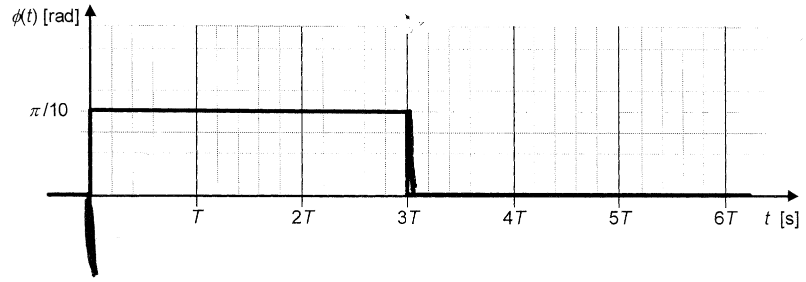
- What is the cutoff frequency (-3 dB point) [Hz] of the slave clock?

$f_c \simeq 850$ Hz

- The input timing signal $s_{IN}(t)$ exhibits a phase error $\phi_{IN}(t)$ vs. the Total Phase of the ideal timing signal $s(t)$ as shown in the graphs below. Plot on the same graph the phase error $\phi_{OUT}(t)$-$\phi_{IN}(t)$. Explain your responses and express your considerations.



$T = 1$ ms



$T = 1$ s

6) What is the *jitter tolerance* of a digital network interface? How is it defined? Hint a method to measure it on an equipment under test. *(4 points)*