Optical and Transport Networks

Prof. Stefano Bregni **II Exam 2022-23 – 10 February 2023**

Last and first name: *(capital letters)*

(signature)

Matriculation number:

NB: In any exercise, any answer not justified adequately, even with few words, will not be considered.

Problem 1

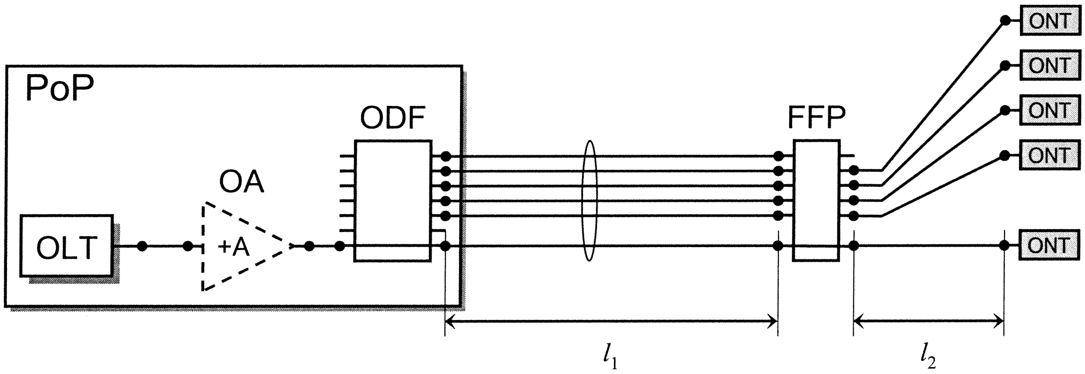
(Solve on this sheet in the space provided) (6 points)

Consider a Point-to-Point (P2P) network reaching 2000 users at variable distances from the Ethernet Optical Line Termination (OLT) according to the scheme in figure.

The line from the OLT is cross-connected via an Optical Distribution Frame (ODF) to the user lines. An Optical Amplifier (OA), if needed, may be added before the ODF at the Point-of-Presence (PoP). After a first feeder fibre segment with length l_1, another ODF (Fibre Flexibility Point, FFP) cross-connects to the users. The fibre segments between the FFP and the users have variable length in the range specified below. The length of other segments of fibres connecting network elements is negligible.

Assume the following data for the P2P network elements:

- fibre with attenuation $\alpha = 0.4$ dB/km;
- $l_1 = 15$ km, 50 m $\leq l_2 \leq$ 10 km;
- OLT transmission power P_{TX};
- splitter insertion loss $\alpha_s = 1$ dB;
- power loss by each couple of optical connectors $\alpha_c = 0.5$ dB (connections marked with dots in figure);
- sensitivity of ONT receivers $P_{RX} > -33$ dBm, with at least 6 dB of safety margin to be guaranteed;
- optional OA gain $+A$ [dB] (excluding the additional attenuation $2\alpha_c$ introduced by its two couples of connectors);



a) Evaluate the maximum *Differential Path Loss* [dB] between ONTs.

b) Evaluate the minimum OLT transmission power P_{TX} (in [dB] and [W]) necessary to reach the farthest ONT (without OA).

c) If the OLT transmission power is P_{TX} = -7 dBm and there is no OA, what is the maximum total distance $L = l_1 + l_2$ between the PoP and the users that can be covered?

d) The same P2P network is used to connect base stations of a cellular network. What is the maximum Time Alignement Error between any couple of terminals (ONT) synchronized by the received signals, assuming these signals are synchronous? Explain your calculation. How would you compensate such TAE?

a) $\Delta PL = \Delta l \cdot \alpha = (9.95\ \text{km}) \cdot (0.4\ \text{dB/km}) = 3.98\ \text{dB}$

b) $P_{TX} \geq -33\ \text{dBm} + 6\ \text{dB} + 6\alpha_c + (l_1 + l_{2max})\,\alpha = -14\ \text{dBm} = 39.8\ \mu W$

c) $(l_1 + l_2)\,\alpha < 17\ \text{dB} \rightarrow L < 42.5\ \text{km}$

d) $\Delta L = 9.95\ \text{km}$ $TAE = 49.75\ \mu s$

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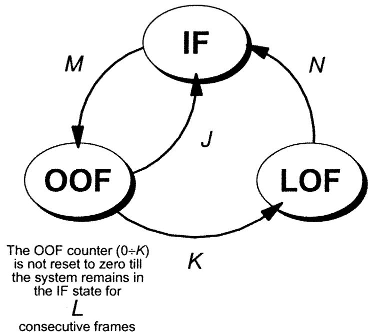
Problem 2

(Solve on this sheet in the space provided) (6 points)

Consider the standard SDH frame alignment algorithm, represented by the diagram below. The frame aligner operates on a test STM-1 framed signal at input (nominal frequency f_0 = 155.520 Mbit/s, frame length L_m = 19440 bit), with random content everywhere in all frames except the alignment word, which consists of X bits during hunting and 8 bits during maintenance. The test signal is affected by random transmission errors on the line, uncorrelated and with rate ε.

a) Let P_1 be the probability that the system, being aligned and in service (IF state), moves to Out-Of-Frame (OOF) within 625 μs due to the random transmission errors. What is the limit value of ε, in order to have $P_1 < 10^{-25}$?



$$P_1 = [1-(1-\varepsilon)^8]^5 \cong (8\varepsilon)^5 < 10^{-25}$$

$$\Rightarrow \varepsilon < 1{,}25 \cdot 10^{-6}$$

b) Let P_2 be the probability that the system, being in state Loss-of-Frame (LOF), gains alignment (IF state) within 3 ms due to simulation of the alignment word by the random payload. Compute the number of bits X, which are necessary to have $P_2 < 10^{-25}$.

$P_2 = \left(\frac{1}{2^X}\right)^{24}$ $\quad 2^{-24X} < 10^{-25}$ $\quad 24X \log 2 > 25$

$\Rightarrow X > 3{,}4$ bit

$X \geq 4$ bit

c) Define and explain the concepts of *forced loss of alignment* vs. *real loss of alignment.*

d) Give a good reason to have N small and another to have N large.

Problem 3

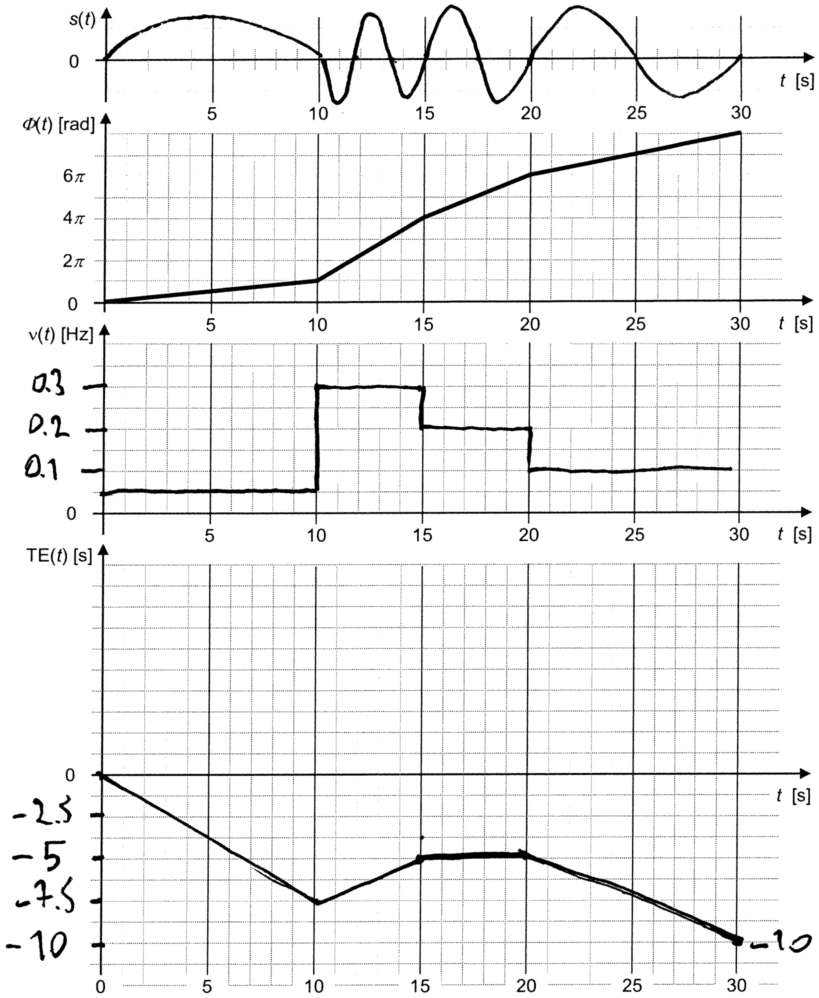
(Solve on this sheet in the space provided) (6 points)

a) Let $s(t)$ be a pseudo-sinusoidal timing signal with *Total Phase* $\Phi(t)$ as plotted in figure.

- Calculate the average frequency of $s(t)$ over the interval $0 \le t \le 30$ s.

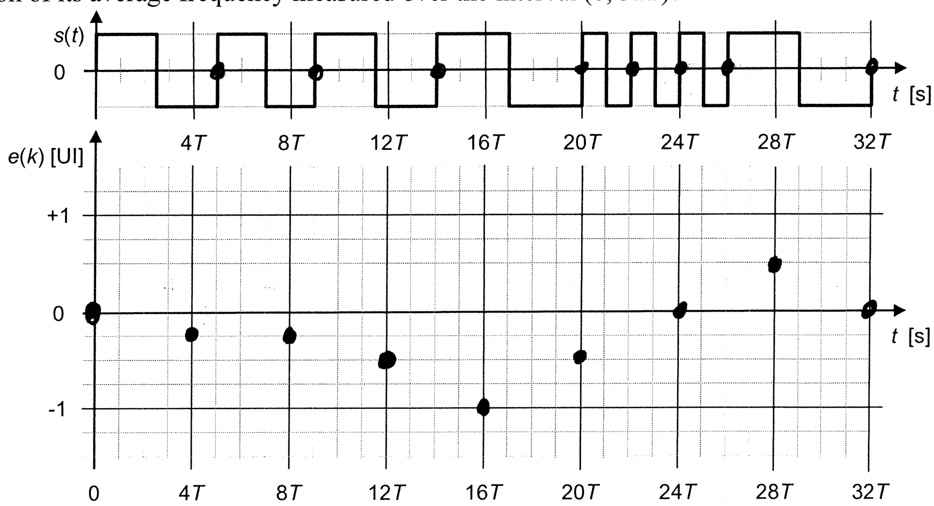
Where possible, plot on the graphs at right:

- the timing signal $s(t)$;
- the *instantaneous frequency* $\nu(t)$;
- the *Time Error* TE(t) with respect to an ideal reference timing signal with frequency $\nu_0 = 0.2$ Hz, starting from TE(0)=0, with the convention that positive TE denotes time advance.



b) Let $s(t)$ be the square timing signal $s(t)$ plotted in figure for $0 \le t \le 32T$ and with nominal frequency $\nu_0 = 1/(4T)$ Hz.

- What is the fractional deviation of its average frequency measured over the interval $(0, 32T)$?
- Plot on the graph the jitter values $e[k]$ measured in [UI], at significant instants $t_k = k(4T)$ of the ideal timing signal with frequency ν_0, starting from the initial point $e[0] = 0$, with the convention that positive jitter denotes time advance.



$$\frac{\Delta V}{V_0} = 0$$

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Problem 4

(Solve on this sheet in the space provided) (6 points)

Let $s(t)$ be a non-ideal timing signal generated by a clock with initial instantaneous frequency set to the nominal frequency $\nu(0) = \nu_0 = 1$ MHz and coefficient of linear frequency drift $D = 10^{-8}$/day.

a) Derive the analytical expression of its *Time* T(t) (where t [days]) knowing that $\Phi(0) = 0$.

$$\nu(t) = \nu_0 + Dt\nu_0 = \nu_0(1 + 10^{-8}t)$$

$$T(t) = t + D\frac{t^2}{2}$$

t [d]

$$\Phi(t) = 2\pi \int_0^t \nu(t)\,dt$$

$$T(t) = \frac{\Phi(t)}{2\pi\nu_0}$$

b) Under the assumption that the frequency drift remains linear with coefficient D indefinitely, evaluate the *Time Error* TE(t) [s] measured by this clock at $t = 1$ year (365 days) with respect to an ideal timing signal with constant frequency ν_0 and same phase at $t = 0$.

$$T(t) = t + D\frac{t^2}{2}$$

$$TE(t) = T(t) - t = \frac{D}{2}t^2$$

For $t = 365$ days:

$$TE(t) = \frac{1}{2} 10^{-8}/\text{day} \cdot (365^2 \text{ days}^2) =$$

$$= 57{,}55 \text{ sec}$$

c) If the frequency drift remains linear with coefficient D indefinitely, after how ~~long~~ many days the *Time Error* measured by this clock would be 1 s?

$$TE(t) = 1 \text{ sec} = \frac{1}{86400} \text{ days}$$

$$\frac{D}{2}t^2 = \frac{1}{86400} \rightarrow t = 48{,}11 \text{ days}$$

Problem 5

(Answer on this sheet in the space provided) (12 points)
NB: In any exercise, any answer not justified adequately, even with few words, will not be considered.

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1) Consider an STM-16 signal transmitted over an optical fibre with refractive index $n = 1.45$ and length $L = 500$ km. Knowing that the coefficient of fractional variation of length vs. temperature of the fibre is $\frac{1}{l}\frac{\partial l}{\partial \theta} = +8{\cdot}10^{-6}$/K, calculate the peak-to-peak amplitude (expressed in [UI]) of the wander caused by fibre length variation induced by 20°C diurnal excursion of fibre temperature. *(2 points)*

$$\tau = \frac{1}{r} = L\frac{n}{c} \rightarrow \Delta\tau = \Delta L \cdot \frac{n}{c}$$

$$\Delta L = +l\left(\frac{1}{l}\frac{\partial l}{\partial \theta}\right)\Delta\theta = (500\ \text{km})\cdot(0{,}8\cdot 10^{-6}/\text{K})\cdot(20\ \text{K}) = 8\ \text{m}$$

$$\Delta\tau = 38.7\ \text{ns} = 96.2\ \text{UI}$$

$$f_{STM\text{-}16} = 2.48832\ \text{Gb/s}$$

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2) Why a VC-4 can be used, but is not well suitable, to transport a 100M-Ethernet signal? What structure of VCs would you use, instead? *(2 points)*

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3) What is the specification of *jitter tolerance* at an equipment digital interface? *(2 points)*

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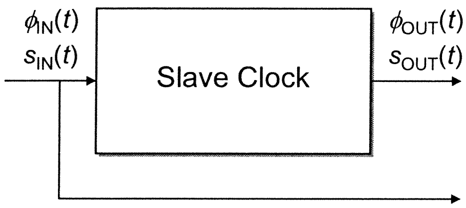
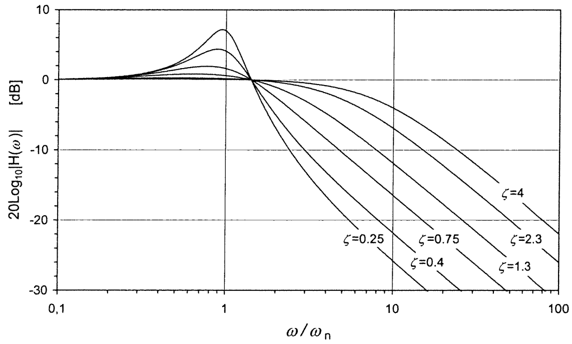
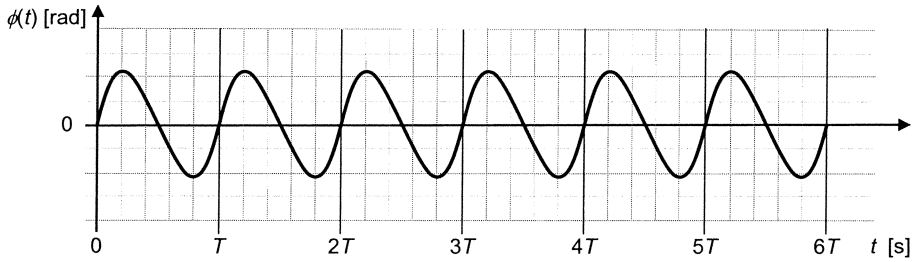
4) Consider a Slave Clock based on a *second-order PLL* system, as shown in the figure below at left. Let us denote as $s_{IN}(t)$ and $s_{OUT}(t)$ its input and output timing signals, respectively, and as $\phi_{IN}(t)$ and $\phi_{OUT}(t)$ their respective *phase errors* vs. the Total Phase of the ideal timing signal $s(t) = A \sin 2\pi\nu_0 t$ considered as common reference in this model, having frequency ν_0. Therefore: $s_{IN}(t) = A \sin(2\pi\nu_0 t + \phi_{IN}(t))$ and $s_{OUT}(t) = A \sin(2\pi\nu_0 t + \phi_{OUT}(t))$. The closed-loop transfer function of the PLL is the standard $H(s)$ plotted below at right ($\omega_n = \pi\ \mathrm{s}^{-1}$, $\zeta = 0.1$). *(3 points)*

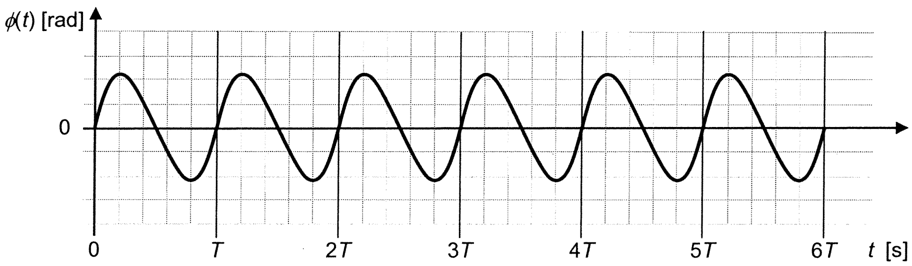




The input timing signal $s_{IN}(t)$ exhibits a phase error $\phi_{IN}(t)$ vs. the Total Phase of the ideal timing signal $s(t)$ as shown in the graphs below (sinusoidal). Plot on the same graphs the output phase $\phi_{OUT}(t)$ ignoring the initial phase alignment between input and output. Express your considerations.



$T = 2$ ms



$T = 2$ s

5) Define the efficiency η of bit error rate estimation by a BIP code. Explain why it is a decreasing function of the line bit error rate ε, from $\eta \to 1$ (for $\varepsilon \to 0$) to $\eta \to 0$. Why consecutive errors may affect negatively the efficiency of bit error rate estimation? How BIP codes cope with that problem? *(3 points)*