Optical and Transport Networks

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Last and first name: *(capital letters)*

(signature)

Matriculation number:

NB: In any exercise, any answer not justified adequately, even with few words, will not be considered.

Problem 1

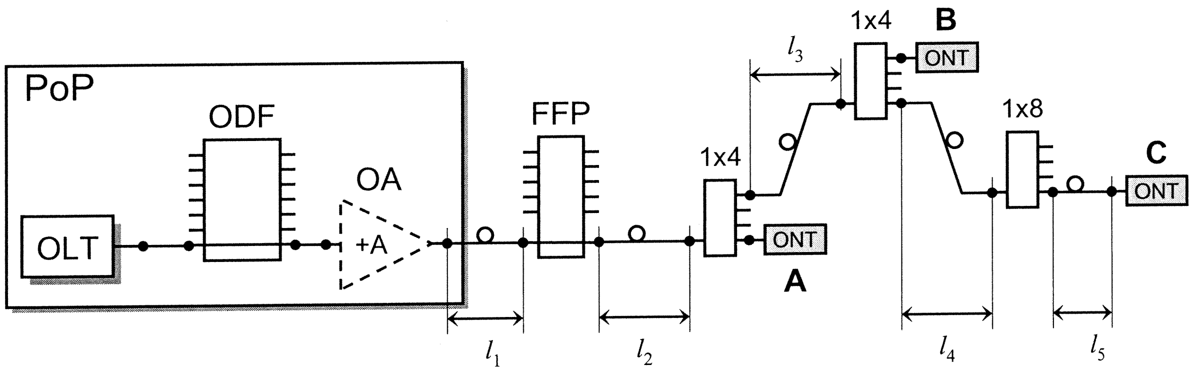
(Solve on this sheet in the space provided) (6 points)

Consider a Passive Optical Network reaching up to N users at variable distances from the Optical Line Termination (OLT) via a variable number of 1×4 and 1×8 splitters, with an asymmetric tree topology according to the scheme in figure.

The line from the OLT is cross-connected via an Optical Distribution Frame (ODF) to the PON. An Optical Amplifier (OA), if needed, may be added after the ODF at the Point-of-Presence (PoP). After a first single feeder fibre segment with length l_1, another ODF (Fibre Flexibility Point, FFP) cross-connects to the PON. The fibre segments between the FFP and the following splitters have length l_2, l_3, l_4, l_5, respectively. The length of other segments of fibres connecting network elements is negligible. The Optical Network Terminations (ONT) can be connected at the output of any splitter at the three stages (A, B, C).

Assume the following data for the PON elements:

- fibre with attenuation $\alpha = 0.3$ dB/km;
- $l_1 = 2$ km, $l_2 = 2$ km, $l_3 = 1$ km, $l_4 = 1$ km, $l_5 = 5$ km;
- OLT transmission power $P_{TX} = -2$ dBm;
- splitter insertion loss $\alpha_s = 1$ dB;
- power loss by each couple of optical connectors $\alpha_c = 0.5$ dB (connections marked with dots in figure);
- sensitivity of ONT receivers $P_{RX} > -33$ dBm, with at least 6 dB of safety margin to be guaranteed;
- optional OA gain $+A$ [dB] (excluding the additional attenuation $2\alpha_c$ introduced by its two couples of connectors);



a) Evaluate the maximum *Differential Path Loss* [dB] between ONTs.

b) Evaluate the power P_{RX} (in [dB] and [W]) received by the farthest ONT in position C without OA.

c) Determine if it is necessary to add an OA, to make the power P_{RX} received by the farthest ONT not less than the minimum power required at the ONT receiver.
   - If the OA is necessary, calculate the minimum OA gain (excluding the additional attenuation $2\alpha_c$ introduced by its two couples of connectors) required.
   - Otherwise, if the system is feasible without OA, calculate the maximum length L of the last fiber segment, to have P_{RX} at any ONT not less than the sensitivity of receivers including the safety margin.

d) The OLT transmits a timing signal downstream to synchronize all ONTs. What is the maximum Time Error between any couple of ONTs synchronized by the received signal? Explain your calculation.

a) $\Delta PL = P_{RX/A} - P_{RX/C} = \alpha(l_3 + l_4 + l_5) + 5\alpha_c + (\alpha_s + 6) + (\alpha_s + 9) =$ [dB]

$= 21,6$ dB

b) $P_{RX/C} = P_{TX} - 12 \cdot \alpha_c - (\alpha_s + 9) - 2(\alpha_s + 6) - \alpha \sum_1^5 l_i$ [dB] $= -35.3$ dBm

$= 0,295\ \mu W$

c) $P_{RX/C} < -27$ dBm $\Rightarrow$ OA necessary $A \geq 8,3$ dB $+ 2\alpha_c = 9,3$ dB

d) $\max(\Delta l) = l_3 + l_4 + l_5 = 7$ km $\Delta\tau = TE = 35\ \mu s$

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Problem 2

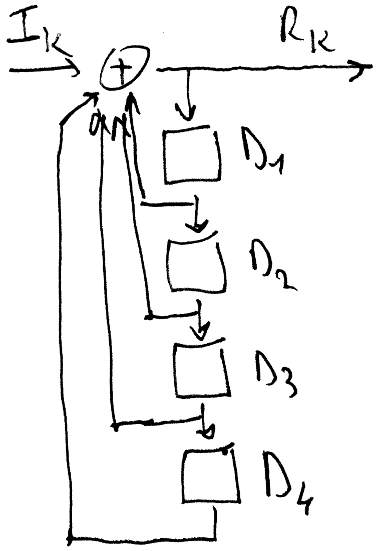
(Solve on this sheet in the space provided) (6 points)

a) Draw the scheme of a *self-synchronizing scrambler* with characteristic polynomial $P(x) = 1 + x + x^2 + x^3 + x^4$, fed with all "0"s and utilized as PRBS generator. Denote the bit sequence at the input as $\{I_k\} = \{0, 0, \ldots\}$ and the sequence at the output as $\{R_k\}$.

b) Initialize the delay cells D_i ($i = 1, 2, 3, 4$) as $\{1, 0, 0, 0\}$ at the initial step $k = 0$. Calculate the PRBS sequence $\{R_k\}$ generated at the output, highlighting its periodicity. What is its period P?

c) What is the maximum period that we could expect from this scrambler with this characteristic polynomial? What are the values of the period, which are possible for a scrambler with characteristic polynomial with grade 4 as this one? Is $P(x)$ reducible or irreducible?

a)



b)

k	I_k	D_{1k}	D_{2k}	D_{3k}	D_{4k}	R_k
0	0	1	0	0	0	1
1	0	1	1	0	0	0
2	0	0	1	1	0	0
3	0	0	0	1	1	0
4	0	0	0	0	1	1
5	0	1	0	0	0	1

$P = 5$

c) $P \leq 2^4 - 1 = 15$

$P(x)$ divisibile by $(x+1)$? NO

by (x^2+x+1)? NO

$\Rightarrow$ irreducibile

$\Rightarrow P \backslash 15 \quad P \in \{1, 3, 5, 15\}$

$$\begin{array}{l|l} x^4+x^3+x^2+x+1 & x+1 \\ \underline{x^4+x^3} & \overline{x^3+x} \\ \quad x^2+x+1 & \\ \quad \underline{x^2+x} & \\ \qquad 1 & \end{array}$$

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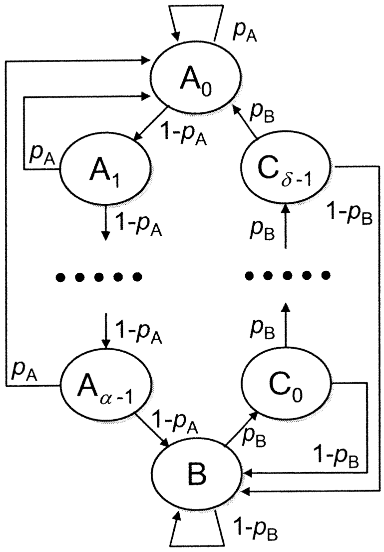
Problem 3

(Solve on this sheet in the space provided) (6 points)

Consider the frame alignment algorithm represented by the diagram below (A_0 state: system aligned in service; B state: alignment lost). The frame aligner operates on a test PDH E3 framed signal at input (nominal frequency $f_0 = 34.368$ Mbit/s, frame length $L_m = 1536$ bit), with random content everywhere in all frames except the alignment word (10 bits during both hunting and maintenance). The test signal is affected by random transmission errors, uncorrelated and with rate ε. According to ITU-T Rec. G.754, for E3 the standard values of the frame alignment parameters are $\alpha = 3$, $\delta = 2$.

a) Let the system be aligned and in service (A_0 state). What are the limit values of α and δ, in order to have the probability of *forced loss of alignment* less than 10^{-15}, if the line bit error rate is $\varepsilon = 10^{-8}$?



$P_A = (1-\varepsilon)^{10} \cong 1-10\varepsilon$

$P_{FL} = (1-P_A)^{\alpha} < 10^{-15}$

$(10\varepsilon)^{\alpha} < 10^{-15}$

$\alpha > \frac{15}{7}$ $\alpha \geq 3$

b) Let the system be out of alignment (B state). What are the limit values of α and δ, in order to have the probability of *fake alignment* less than 10^{-15}?

$P_B = \frac{1}{2^{10}} = 9{,}765 \cdot 10^{-4}$ $P_B^{\delta+1} < 10^{-15}$ $(\delta+1)\log_{10} P_B < -15$

$P_{FA} = P_B^{\delta+1}$

$\Rightarrow \delta > 3{,}98 \Rightarrow \delta \geq 4$

c) What would be the disadvantage, should we set α and δ twice as standard values $\alpha = 3$, $\delta = 2$?

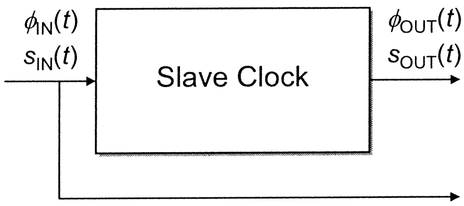
Problem 4

(Solve on this sheet in the space provided) (6 points)

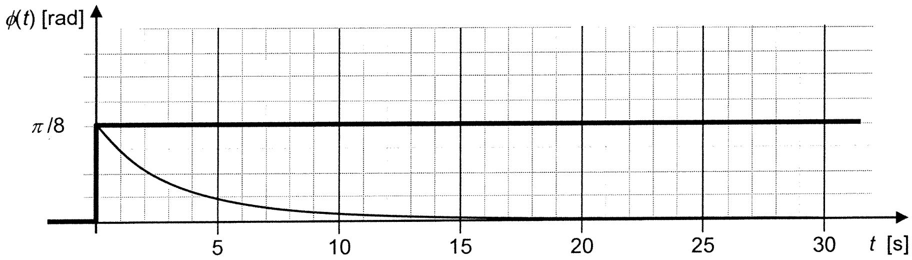
Consider a Slave Clock based on a second-order PLL system, as shown in the figure below.

Let us denote as $s_{IN}(t)$ and $s_{OUT}(t)$ its input and output timing signals, respectively, and as $\phi_{IN}(t)$ and $\phi_{OUT}(t)$ their respective phase errors vs. the Total Phase of the ideal timing signal $s(t) = A \sin 2\pi\nu_0 t$ considered as common reference in this model, having frequency ν_0. Therefore: $s_{IN}(t) = A \sin (2\pi\nu_0 t + \phi_{IN}(t))$ and $s_{OUT}(t) = A \sin (2\pi\nu_0 t + \phi_{OUT}(t))$.

The phase step amplitude $\pi/8$ is supposed small enough to make the linear model a valid approximation for the PLL behaviour. The closed-loop transfer function of the PLL is therefore the standard $H(s)$ with 1 zero and 2 poles. Let its time constant $T = 1/B$.



If a phase step $\phi_{IN}(t)$ as shown in the graph below is applied on the input timing signal $s_{IN}(t)$, then the output-input phase error $\Delta\phi(t) = \phi_{OUT}(t) - \phi_{IN}(t)$ is measured as the curve line in the graph below.



a) Explain the behaviour of the system and of the shape of the curve $\Delta\phi(t)$. In particular, explain why $\Delta\phi(t)$ decreases to 0 with time.

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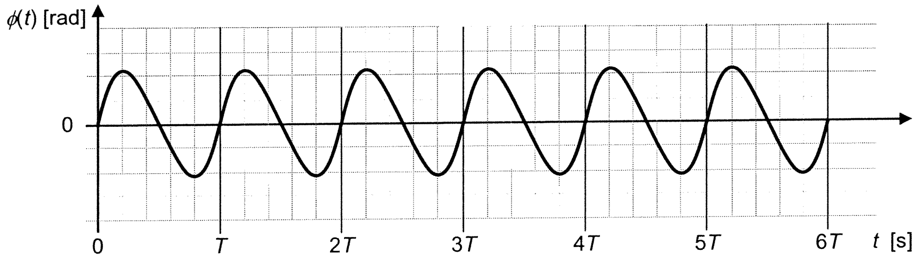
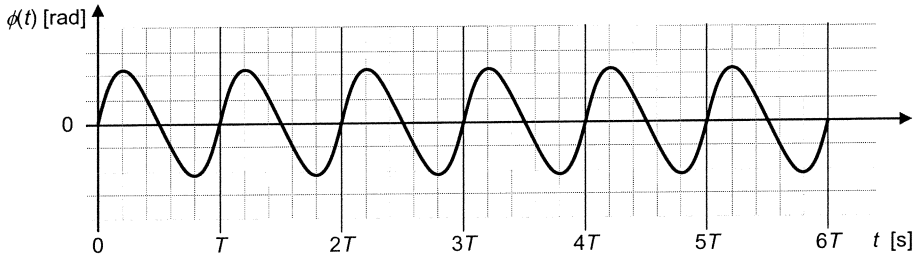
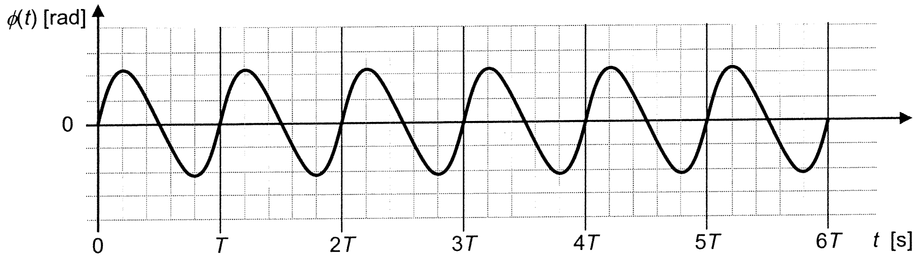
b) The input timing signal $s_{IN}(t)$ exhibits a phase error $\phi_{IN}(t)$ vs. the Total Phase of the ideal timing signal $s(t)$ as shown in the graphs below. Plot on the same graphs the output phase $\phi_{OUT}(t)$ in the three cases for T = 1 ms, 1 s, 1000 s, ignoring the initial phase alignment between input and output. If in some case you are not sure about the plot, specify what additional information you would like to know about this PLL.



T = 1 ms



T = 1 s



T = 1000 ms

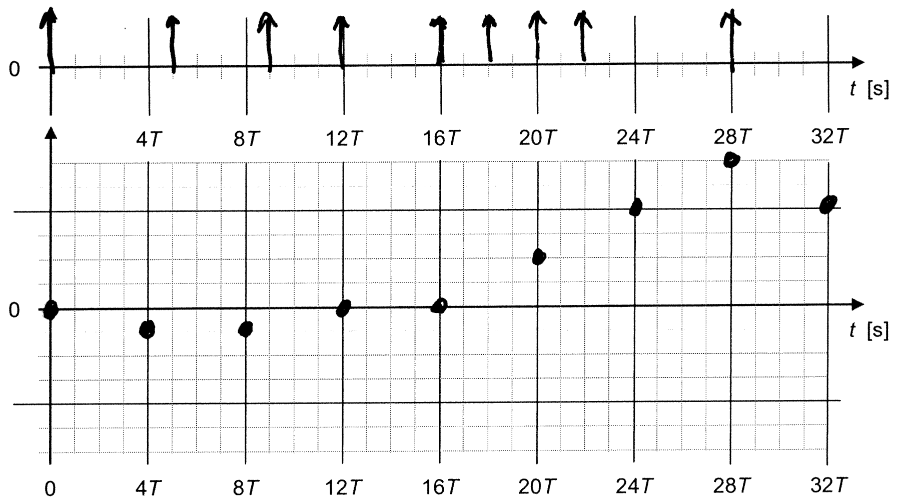
Explain your considerations.

Problem 5

(Answer on this sheet in the space provided) (12 points)

NB: In any exercise, any answer not justified adequately, even with few words, will not be considered.

1) A source transmits packets to a destination with constant rate every $4T$. Packets are supposed short enough to have duration negligible compared to T. Nine packets numbered $k = 0, 1, \ldots, 8$ are transported over the network and arrive to their destination with the sequence of inter-arrival times $\{y_k\} = (5T, 4T, 3T, 4T, 2T, 2T, 2T, 6T)$, where y_k is the inter-arrival time between packet k and the next one.
Plot on the graph the PDV values $e[k]$, measured in T units, at the instants $t_k = k(4T)$ of ideal arrival of packets, besides the latency of packet 0, starting from the initial point $e[0] = 0$, with the convention that positive PDV denotes time advance. *(2 points)*



2) In a PON, the guard time between bursts transmitted by ONTs has been set to $\Delta T = 10$ µs. What is the maximum fractional frequency error $\Delta\nu/\nu_0$ between ONTs, should they be unsynchronized, in order to not have more than one collision of bursts at the OLT every day? *(2 points)*

$$\frac{\Delta\nu}{\nu_0} T < \Delta T \qquad \frac{\Delta\nu}{\nu_0} < \frac{\Delta T}{T} = 1{,}15\cdot 10^{-10}$$

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3) Define the efficiency η of bit error rate estimation by a BIP code. Explain why it is a decreasing function of the line bit error rate ε, from $\eta \to 1$ (for $\varepsilon \to 0$) to $\eta \to 0$. Why consecutive errors may affect negatively the efficiency of bit error rate estimation? How BIP codes cope with that problem? *(3 points)*

4) What are the main disadvantages of a P2P Ethernet solution vs. PON, in deploying an FTTX access system in a urban area? *(2 points)*

5) Outline a procedure to measure the *jitter tolerance* at a STM-1 interface of transmission equipment. List any instrumentation you may need to carry out this test. *(3 points)*