Optical and Transport Networks

Prof. Stefano Bregni **IV Exam 2022-23 – 11 July 2023**

Last and first name: *(capital letters)*

(signature)

Matriculation number:

NB: In any exercise, any answer not justified adequately, even with few words, will not be considered.

Problem 1

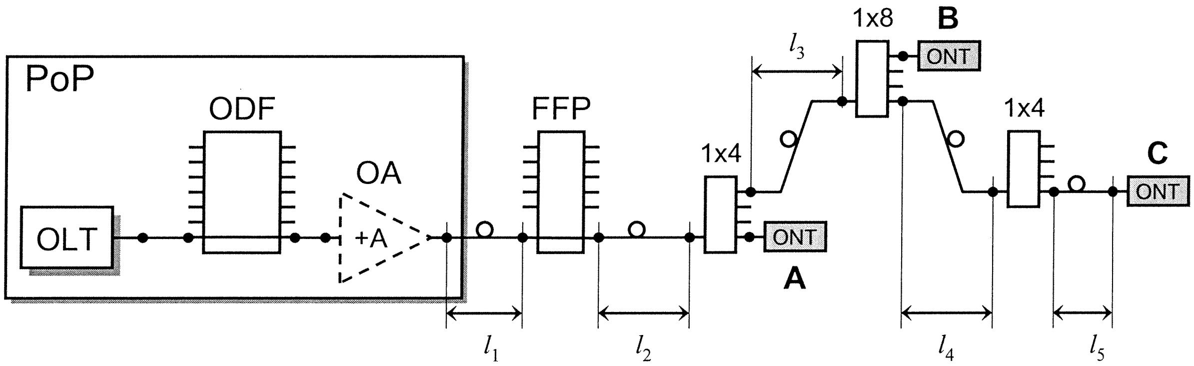
(Solve on this sheet in the space provided) (6 points)

Consider a Passive Optical Network reaching up to N users at variable distances from the Optical Line Termination (OLT) via a variable number of 1×4 and 1×8 splitters, with an asymmetric tree topology according to the scheme in figure.

The line from the OLT is cross-connected via an Optical Distribution Frame (ODF) to the PON. An Optical Amplifier (OA), if needed, may be added after the ODF at the Point-of-Presence (PoP). After a first single feeder fibre segment with length l_1, another ODF (Fibre Flexibility Point, FFP) cross-connects to the PON. The fibre segments between the FFP and the following splitters have length l_2, l_3, l_4, l_5, respectively. The length of other segments of fibres connecting network elements is negligible. The Optical Network Terminations (ONT) can be connected at the output of any splitter at the three stages (A, B, C).

Assume the following data for the PON elements:

- fibre with attenuation $\alpha = 0.6$ dB/km;
- $l_1 = 2$ km, $l_2 = 2$ km, $l_3 = 1$ km, $l_4 = 1$ km, $l_5 = 1$ km;
- OLT transmission power $P_{TX} = 100$ μW;
- splitter insertion loss $\alpha_s = 1$ dB;
- power loss by each couple of optical connectors $\alpha_c = 0.5$ dB (connections marked with dots in figure);
- sensitivity of ONT receivers $P_{RX} > -33$ dBm, with at least 6 dB of safety margin to be guaranteed;
- optional OA gain $+A$ [dB] (excluding the additional attenuation $2\alpha_c$ introduced by its two couples of connectors);



a) Evaluate the maximum *Differential Path Loss* [dB] between ONTs.

b) Evaluate the power P_{RX} [W] received by the farthest ONT in position C without OA.

c) Determine if it is necessary to add an OA, to make the power P_{RX} received by the farthest ONT not less than the minimum power required at the ONT receiver.
   - If the OA is necessary, calculate the minimum OA gain (excluding the additional attenuation $2\alpha_c$ introduced by its two couples of connectors) required.
   - Otherwise, if the system is feasible without OA, calculate the maximum length L of the last fiber segment, to have P_{RX} at any ONT not less than the sensitivity of receivers including the safety margin.

d) The OLT transmits a square timing signal downstream to synchronize all ONTs, which are equipped with PLL-based slave clocks. Can ONTs be synchronized on the same absolute time? Their relative *Time Error*, between any pair, will constant or variable, in absence of any jitter?

a) $\Delta PL = P_{RX/A} - P_{RX/C} = \alpha(l_3 + l_4 + l_5) + 5\alpha_c + (\alpha_s + 9) + (\alpha_s + 6) =$

$= 21{,}3\ dB$

b) $P_{RX/C} = P_{TX} - 12\alpha_c - (\alpha_s + 9) - 2(\alpha_s + 6) - \alpha \sum_{1}^{5} l_i = \quad [dB]$

$= -44{,}2\ dBm$

$= 38\ nW$

c) $P_{RX/C} < -27\ dBm \Rightarrow \exists A$

$A \geq 17.2\ dB + 2\alpha_c = 18.2\ dB$

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Problem 2

(Solve on this sheet in the space provided) (6 points)

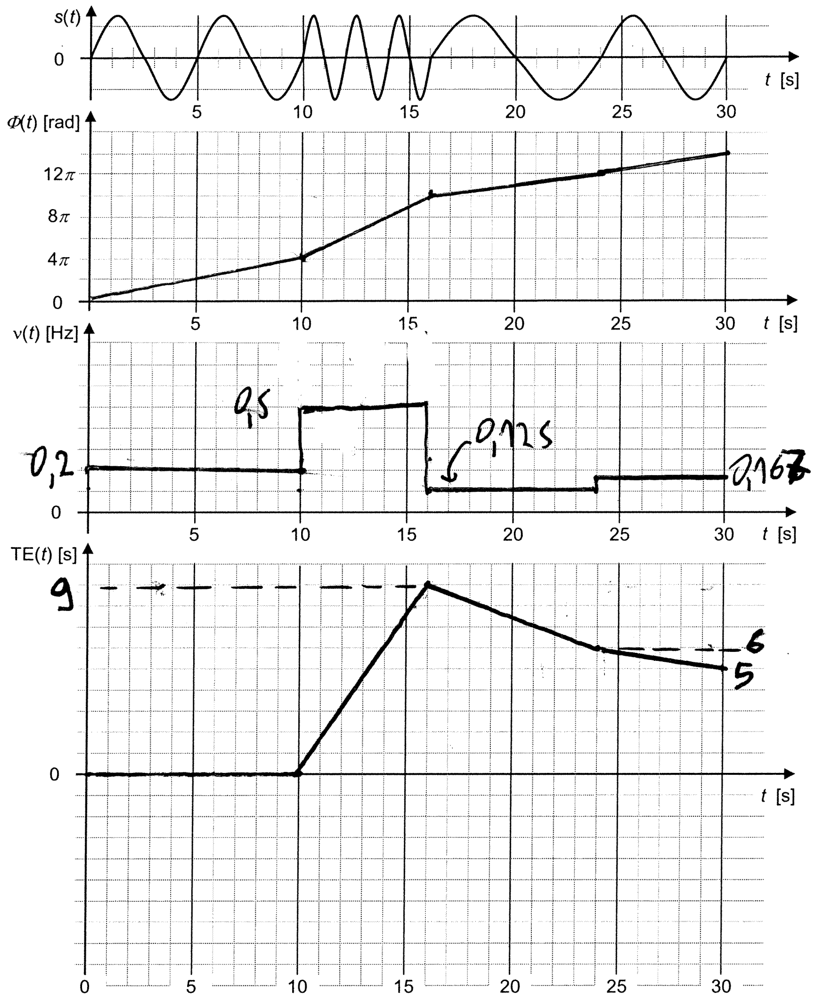
a) Let $s(t)$ be a pseudo-sinusoidal timing signal as plotted in figure.

- Calculate the average frequency of $s(t)$ over the interval $0 \le t \le 30$ s.

Where possible, plot on the graphs at right:

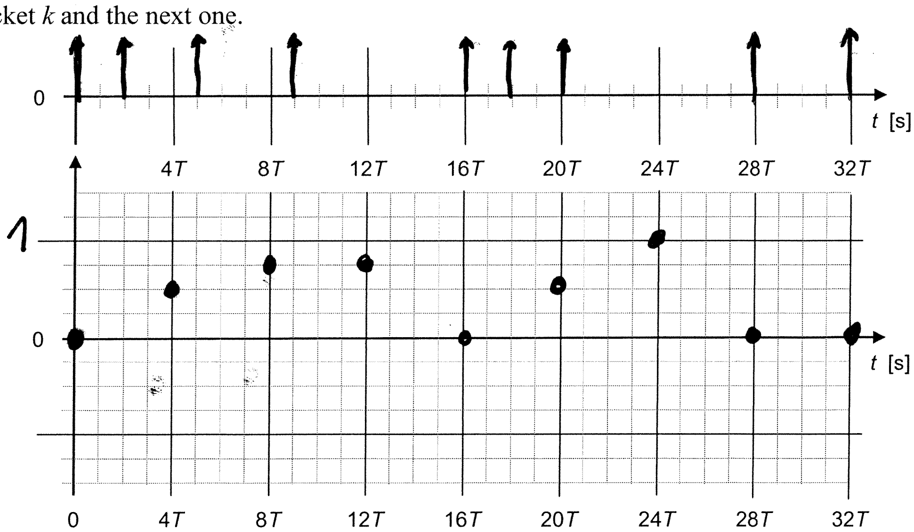
- the *Total Phase* $\Phi(t)$;
- the *instantaneous frequency* $\nu(t)$;
- the *Time Error* TE(t) with respect to an ideal reference timing signal with frequency $\nu_0 = 0.2$ Hz, starting from TE(0)=0, with the convention that positive TE denotes time advance.

$$\overline{\nu(t)} = \frac{7\ \text{cycles}}{30\ \text{s}} = 0{,}23\bar{3}\ \text{Hz}$$



b) A source transmits packets to a destination with constant rate every $4T$. Packets are supposed short enough to have duration negligible compared to T. Nine packets numbered $k = 0, 1, \ldots, 8$ are transported over the network and arrive to their destination with the sequence of inter-arrival times $\{y_k\} = (2T, 3T, 4T, 7T, 2T, 2T, 8T, 4T)$, where y_k is the inter-arrival time between packet k and the next one.

Plot on the graph the PDV values $e[k]$, measured in T units, at the instants $t_k = k(4T)$ of ideal arrival of packets, besides the latency of packet 0, starting from the initial point $e[0] = 0$, with the convention that positive PDV denotes time advance.



Problem 3

(Solve on this sheet in the space provided) (6 points)

Let $s(t)$ be a non-ideal timing signal generated by a clock with initial instantaneous frequency set to the nominal frequency $\nu(0) = \nu_0 = 1$ Hz and coefficient of quadratic frequency drift $D = 10^{-12}/(\text{day})^2$, that is $\nu(t) = \nu_0 + D\,t^2\,\nu_0$

a) Derive the analytical expression of the *Total Phase* $\Phi(t)$ (where t [days]) knowing that $\Phi(0) = 0$.

$$\Phi(t) = 2\pi\nu_0 \int_0^t (1 + Dt^2)\,dt = 2\pi\nu_0 \left(t + \frac{D}{3}t^3\right)$$

b) Under the assumption that the frequency drift remains quadratic with coefficient D indefinitely, evaluate the *Time Error* TE(t) [s] measured by this clock at $t = 1$ year (365 days) with respect to an ideal timing signal with constant frequency ν_0 and same phase at $t = 0$.

$$T(t) = \frac{\Phi(t)}{2\pi\nu_0} = t + \frac{D}{3}t^3$$

$$TE(t) = T(t) - t = \frac{D}{3}t^3$$

For $t = 365$ days:

$$TE(365\,d) = \frac{1}{3}10^{-12}/\cancel{day^2} \cdot (365)^3 \cancel{days^3}$$

$$= 1.4 \text{ sec}$$

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Problem 4

(Solve on this sheet in the space provided) (6 points)

a) Define what are *forced loss of alignment*, *real loss of alignment*, *fake alignment*. Explain when they occur and how to avoid them.

b) For what reason the frame alignment word should have different length during hunting and maintenance? Discuss pros and cons.

Problem 5

(Answer on this sheet in the space provided) (12 points)

NB: In any exercise, any answer not justified adequately, even with few words, will not be considered.

1) Define the *packet jitter* of a sequence of packets transmitted with constant rate over a network. *(2 points)*

2) Define the *Time Interval Error* of a clock generating the time $T(t)$ with respect to the ideal time t. In what case it could take negative values? *(2 points)*

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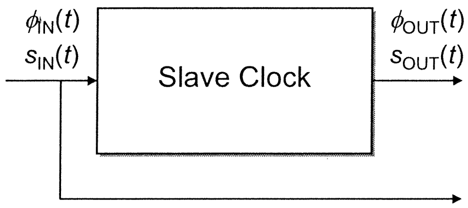
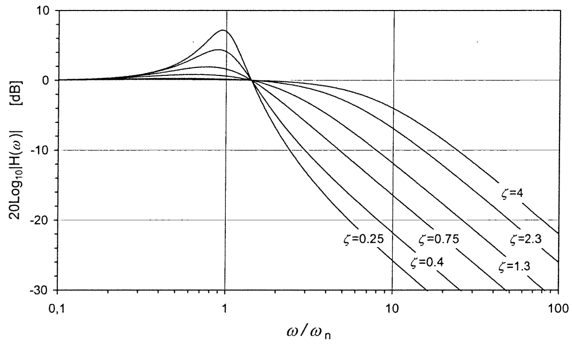
3) What is interleaving in *Bit Interleaved Parity* codes BIP(n,m)? What is its purpose? What is a reasonable criterion to determine the interleaving depth n? *(2 points)*

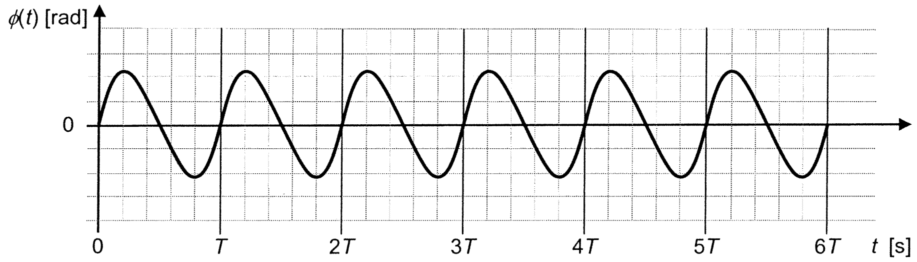
4) Define the efficiency η of bit error rate estimation by a BIP code. Explain why it is a decreasing function of the line bit error rate ε, from $\eta \to 1$ (for $\varepsilon \to 0$) to $\eta \to 0$. Why consecutive errors may affect negatively the efficiency of bit error rate estimation? *(3 points)*

5) Consider a Slave Clock based on a *second-order PLL* system, as shown in the figure below at left. Let us denote as $s_{IN}(t)$ and $s_{OUT}(t)$ its input and output timing signals, respectively, and as $\phi_{IN}(t)$ and $\phi_{OUT}(t)$ their respective *phase errors* vs. the Total Phase $\Phi(t)$ of the ideal timing signal $s(t) = A \sin 2\pi\nu_0 t$ considered as common reference in this model, having frequency ν_0. Therefore: $s_{IN}(t) = A \sin(2\pi\nu_0 t + \phi_{IN}(t))$ and $s_{OUT}(t) = A \sin(2\pi\nu_0 t + \phi_{OUT}(t))$. The closed-loop transfer function of the PLL is the standard $H(s)$ plotted below at right ($\omega_n = 2\pi\ \mathrm{s}^{-1}$, $\zeta = 0.01$). *(3 points)*

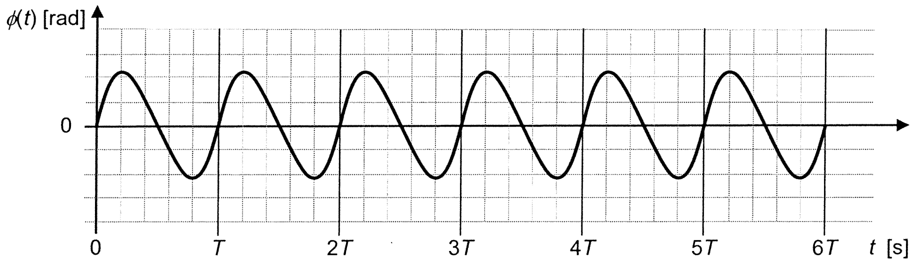




The input timing signal $s_{IN}(t)$ exhibits a phase error $\phi_{IN}(t)$ vs. the Total Phase of the ideal timing signal $s(t)$ as shown in the graphs below (sinusoidal). Plot on the same graphs the output phase $\phi_{OUT}(t)$ ignoring the initial phase alignment between input and output. Express your considerations.



$T = 10$ ms



$T = 100$ s