

Optical and Transport Networks

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Last and first name:

(capital letters)

(signature)

Matriculation number:

NB: In any exercise, any answer not justified adequately, even with few words, will not be considered.

Problem 1

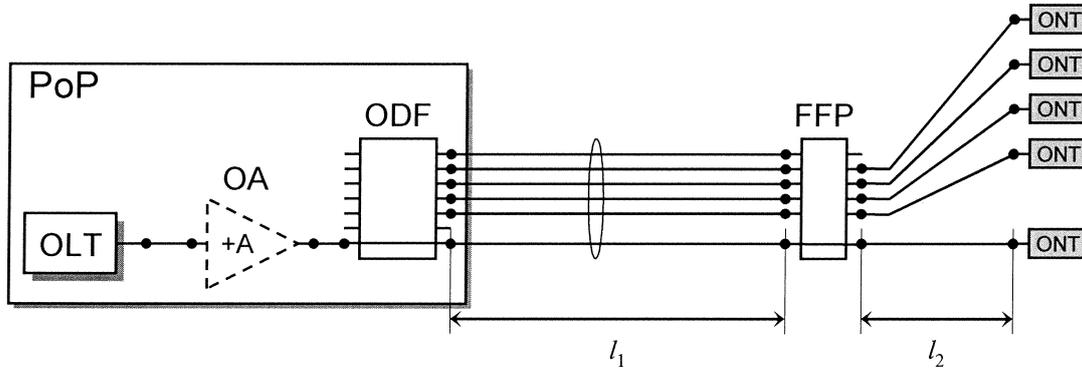
(Solve on this sheet in the space provided) (5 points)

Consider a Point-to-Point (P2P) network reaching 1024 users at variable distances from the Ethernet Optical Line Termination (OLT) according to the scheme in figure.

The line from the OLT is cross-connected via an Optical Distribution Frame (ODF) to the user lines. An Optical Amplifier (OA), if needed, may be added before the ODF at the Point-of-Presence (PoP). After a first feeder fibre segment with length l_1 , another ODF (Fibre Flexibility Point, FFP) cross-connects to the users. The fibre segments between the FFP and the users have variable length in the range specified below. The length of other segments of fibres connecting network elements is negligible.

Assume the following data for the P2P network elements:

- fibre with attenuation $\alpha = 0.2$ dB/km;
- $l_1 = 2$ km, $100 \text{ m} \leq l_2 \leq 20$ km;
- OLT transmission power P_{TX} ;
- splitter insertion loss $\alpha_s = 1$ dB;
- power loss by each couple of optical connectors $\alpha_c = 0.5$ dB (connections marked with dots in figure);
- sensitivity of ONT receivers $P_{RX} > -33$ dBm, with at least 6 dB of safety margin to be guaranteed;
- optional OA gain $+A$ [dB] (excluding the additional attenuation $2\alpha_c$ introduced by its two couples of connectors);



- Evaluate the maximum *Differential Path Loss* [dB] between ONTs.
- Evaluate the minimum OLT transmission power P_{TX} [W] necessary to reach the farthest ONT (without OA).
- Evaluate the total length of fibers [km] deployed to reach 1024 users with the P2P network scheme in figure, if the length of fibers between the FFP and the users is uniformly distributed in the interval $100 \text{ m} \leq l_2 \leq 20$ km.
- What is the maximum total distance $L = l_1 + l_2$ between the PoP and the users that can be covered, if $P_{TX} = 0$ dBm (without OA)?

$$a) \Delta PL = \Delta l \cdot \alpha = (19.9 \text{ km}) \cdot (0.2 \text{ dB/km}) = 3.98 \text{ dB}$$

$$b) P_{TX} \geq -33 \text{ dBm} + 6 \text{ dB} + 6\alpha_c + (l_1 + l_{2\text{MAX}})\alpha = -19.6 \text{ dBm}$$

$$c) 1024 \left(l_1 + \frac{l_{2\text{MIN}} + l_{2\text{MAX}}}{2} \right) = 12.339 \text{ km} = \sim 12.96 \mu\text{W}$$

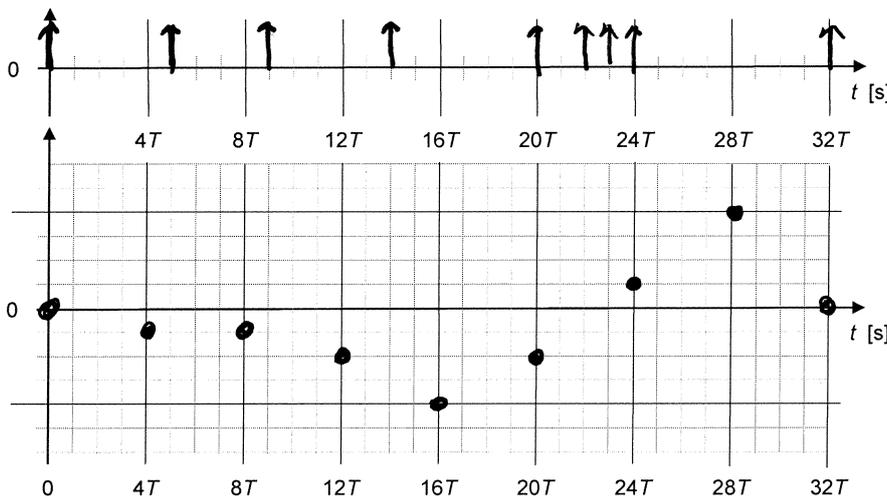
$$d) -27 \text{ dBm} \leq -6\alpha_c - L\alpha \rightarrow L \leq 120 \text{ km}$$

Problem 2

(Solve on this sheet in the space provided) (6 points)

- a) A source transmits packets to a destination with constant rate every $4T$. Packets are supposed short enough to have duration negligible compared to T . Nine packets numbered $k = 0, 1, \dots, 8$ are transported over the network and arrive to their destination with the sequence of inter-arrival times $\{y_k\} = (5T, 4T, 5T, 6T, 2T, T, T, 8T)$, where y_k is the inter-arrival time between packet k and the next one.

Plot on the graph the PDV values $e[k]$, measured in T units, at the instants $t_k = k(4T)$ of ideal arrival of packets, besides the latency of packet 0, starting from the initial point $e[0] = 0$, with the convention that positive PDV denotes time advance.



- b) Outline a procedure to measure the *jitter tolerance* at a STM-1 interface of transmission equipment. List any instrumentation you may need to carry out this test.

Problem 3

(Solve on this sheet in the space provided) (5 points)

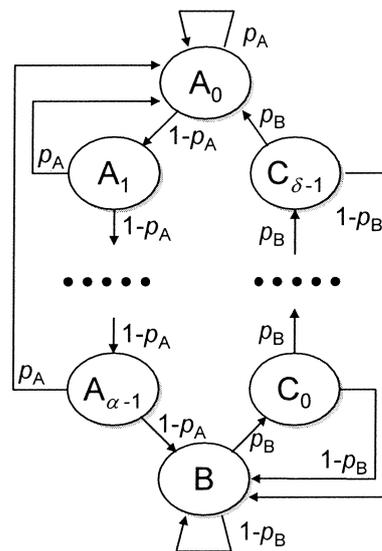
Consider the frame alignment algorithm represented by the diagram below (A_0 state: system aligned in service; B state: alignment lost). The frame aligner operates on a test PDH E4 framed signal at input (nominal frequency $f_0 = 139.264$ Mbit/s, frame length $L_m = 2928$ bit), with random content everywhere in all frames except the alignment word (10 bits during both hunting and maintenance). The test signal is affected by random transmission errors, uncorrelated and with rate ϵ . According to ITU-T Rec. G.754, for the PDH signal E4 the frame alignment parameters are $\alpha = 3, \delta = 2$.

- a) Let the system ($\alpha = 3, \delta = 2$) be aligned and in service (A_0 state). What is the maximum value of the line bit error rate ϵ , in order to have the probability of forced loss of alignment less than 10^{-12} ?

$$P_A = (1 - \epsilon)^{10} \approx 1 - 10\epsilon$$

$$P_{FL} = (1 - P_A)^3 < 10^{-12}$$

$$(10\epsilon)^3 < 10^{-12} \rightarrow \epsilon < 10^{-12/3} = 10^{-4}$$



- b) Let the system be out of alignment (B state). What should be the minimum value of the parameter δ , in order to have the probability of fake alignment less than 10^{-15} ? What is the disadvantage, should we set δ twice as such minimum value?

$$P_B = \frac{1}{2^{10}} = 9.765 \cdot 10^{-4}$$

$$P_B^{\delta+1} < 10^{-15}$$

$$(\delta+1) \log_{10} P_B < -15$$

$$\rightarrow \delta > 3.98 \quad \delta \geq 4$$

- c) Let the system be aligned and in service (A_0 state). For $\varepsilon = 10^{-4}$, what should be the minimum value of the parameter α , in order to have the probability of *forced loss of alignment* less than 10^{-14} ? What is the disadvantage, should we set α twice as such minimum value?

$$P_A \cong 1 - 10\varepsilon$$

$$P_{FL} = (1 - P_A)^\alpha < 10^{-14}$$

$$(10 \cdot 10^{-4})^\alpha < 10^{-14} \quad \alpha(-3) < -14 \quad \alpha \geq \frac{14}{3} \Rightarrow \alpha \geq 5$$

Problem 4

(Solve on this sheet in the space provided) (6 points)

Let $s(t)$ be a non-ideal timing signal generated by a clock with initial instantaneous frequency set to the nominal frequency $\nu(0) = \nu_0 = 1$ Hz and coefficient of linear frequency drift $D = 10^{-9}$ /day.

a) Derive the analytical expression of the *Total Phase* $\Phi(t)$ (where t [days]) knowing that $\Phi(0) = 0$.

$$\nu(t) = \nu_0 + Dt\nu_0 = \nu_0 (1 + 10^{-9}t)$$

$$\Phi(t) = 2\pi\nu_0 \int_0^t (1 + Dt) dt = 2\pi\nu_0 \left(t + D \frac{t^2}{2} \right)$$

b) Under the assumption that the frequency drift remains linear with coefficient D indefinitely, evaluate the *Time Error* $TE(t)$ [s] measured by this clock at $t = 1$ year (365 days) with respect to an ideal timing signal with constant frequency ν_0 and same phase at $t = 0$.

$$T(t) = \frac{\Phi(t)}{2\pi\nu_0} = t + \frac{D}{2}t^2$$

For $t = 365$ days:

$$TE(365 \text{ d}) = \frac{1}{2} 10^{-9} / \text{day} \cdot 365^2 \text{ days}^2 = 5.75 \text{ s}$$

$$TE(t) = T(t) - t = \frac{D}{2}t^2$$

c) If the frequency drift remains linear with coefficient D indefinitely, after how long the *Time Error* measured by this clock would be 1 s?

$$TE(t) = 1 \text{ s} = \frac{1}{86400} \text{ days}$$

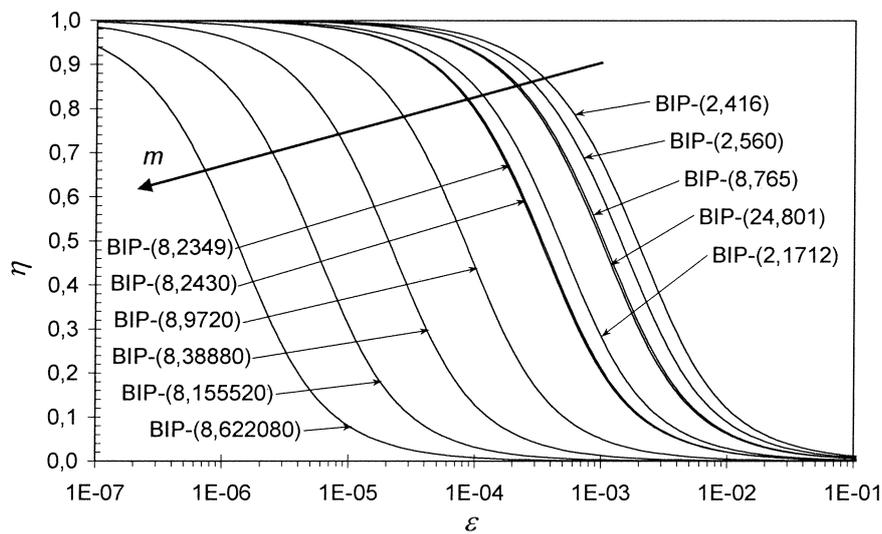
$$\frac{D}{2}t^2 = \frac{1}{86400} \quad t = 152.145 \text{ days}$$

Problem 5

(Answer on this sheet in the space provided) (14 points)

NB: In any exercise, any answer not justified adequately, even with few words, will not be considered.

- 1) Explain what is the efficiency η of bit error rate estimation by a BIP code, plotted below as a function of the actual line bit error rate ε . Why $\eta \rightarrow 1$ for $\varepsilon \rightarrow 0$? Why the plot below, calculated assuming that the line errors are purely random and not correlated over time, can be considered still valid in many practical circumstances, even if errors happen consecutively? (3 points)



- 2) What is the difference between UTC and TAI? (2 points)

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3) Consider an STM-64 signal transmitted over an optical fibre with refractive index $n = 1.5$ and length $L = 300$ km.

Knowing that the coefficient of fractional variation of length vs. temperature of the fibre is $\frac{1}{L} \frac{\partial L}{\partial \theta} = +8 \cdot 10^{-6}/K$,

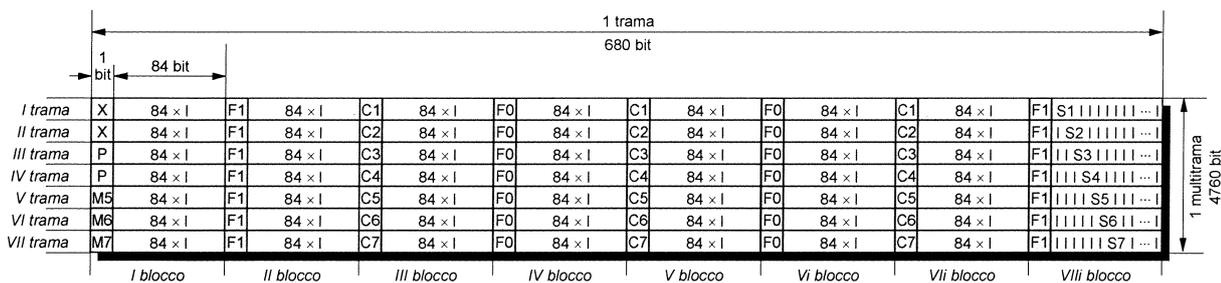
calculate the peak-to-peak amplitude (expressed in [UI]) of the wander caused by fibre length variation induced by 10°C diurnal excursion of fibre temperature. (2 points)

$$v = \frac{L}{n} = L \frac{n}{c} \Rightarrow \Delta v = \Delta L \cdot \frac{n}{c}$$

$$\Delta L = +L \left(\frac{1}{L} \frac{\partial L}{\partial \theta} \right) \Delta \theta = (300 \text{ km}) (8 \cdot 10^{-6} / K) (10 \text{ K}) = 2.4 \text{ m}$$

$$\Delta v = 12 \text{ ns} = 120 \text{ UI}$$

4) The figure below depicts the frame/multiframe structure of the North-American PDH signal DS3 (having nominal frequency $f_0 = 44.736$ Mbit/s), which multiplexes 7 tributaries DS2 ($f_0 = 6.312$ Mbit/s). Calculate the nominal justification ratio (defined as fraction of justification opportunity bits occupied by dummy bits). (2 points)



parola di allineamento di multitrama: M5, M6, M7 = 0, 1, 0
 parola di allineamento di trama: F1, F0, F0, F1 = 1, 0, 0, 1
 X = bit di servizio
 P = bit di parità calcolato sulla multitrama precedente
 I = bit di tributario
 C1, C2, C3, C4, C5, C6, C7 = bit di controllo di giustificazione del tributario 1, 2, 3, 4, 5, 6, 7
 S1, S2, S3, S4, S5, S6, S7 = bit opportunità di giustificazione del tributario 1, 2, 3, 4, 5, 6, 7

Translation notes:
 multitrama = multiframe; trama = frame; blocco = block; parola di allineamento = alignment word;
 tributario = tributary; bit di opportunità di giustificazione = justification opportunity bit

$$\frac{672 - p}{4760} \cdot 44.736 \text{ Mb/s} = 6.312 \text{ Mb/s}$$

$$\rightarrow p = 0,39056$$

- 5) Two Base Stations have relative frequency difference $\Delta\nu/\nu_0 = 5 \cdot 10^{-8}$. At some initial time, they have relative *Time Alignment Error* TAE = 0. After how long will they have cumulated TAE = 100 ns? (2 points)

$$\frac{\Delta T}{T} = 5 \cdot 10^{-8}$$

$$\Delta T = \frac{\Delta\nu}{\nu_0} t = 100 \text{ ns}$$

$$t = 2 \text{ ns}$$

- 6) Data bursts transmitted by users connected to an urban base station of a mobile network are monitored. Initial times and duration of each burst are recorded. What reasons could be mentioned, to suspect that burst start times could not be modeled by a Poisson process? What statistical analysis could you perform on the available data records, to confirm or deny that such model holds or not for those measured data? (3 points)