

# Optical and Transport Networks

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II Exam 2018-19 – 21 February 2019

Last and first name:

(capital letters)

(signature)

Matriculation number:

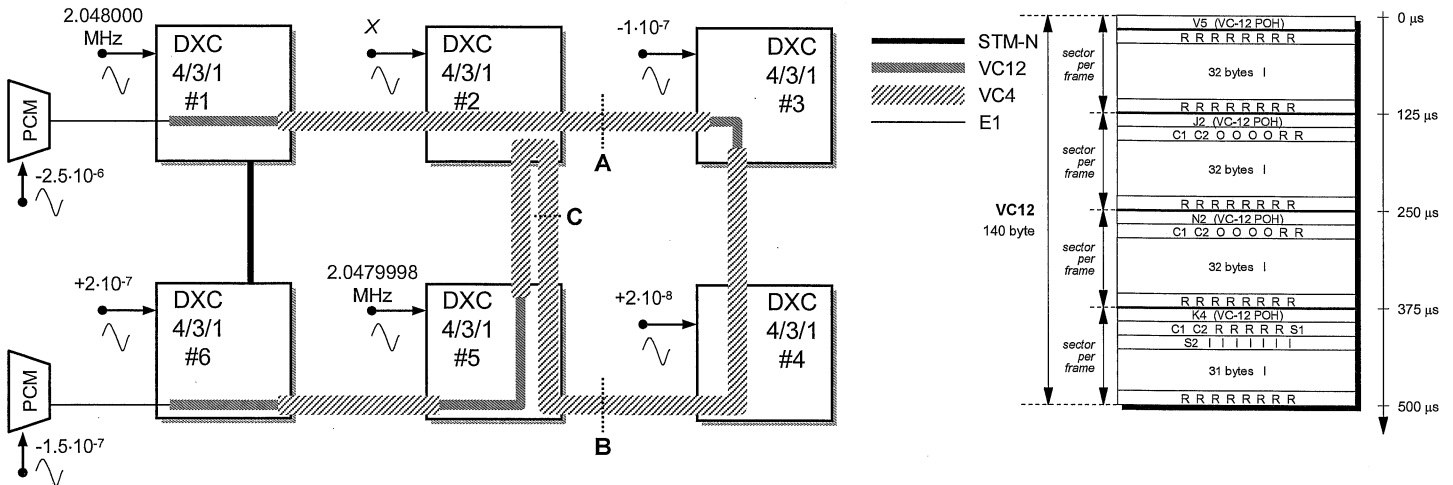
NB: In any exercise, any answer not justified adequately, even with few words, will not be considered.

## Problem 1

(Solve on this sheet in the space provided) (6 points)

Consider the network of DXC 4/3/1 elements in figure below, where all links are bidirectional. Links between DXCs are STM-4 ( $f_0 = 622.080$  Mbit/s). Each Network Element (NE) is synchronized by an external reference, of which either the fractional frequency deviation from the nominal value  $f_0 = 2.048$  MHz or directly the absolute frequency is given. For your convenience, the asynchronous mapping scheme of E1 into VC12 is also given.

Two PCM multiplexers, synchronized by autonomous references, are connected by a bidirectional E1 link ( $f_0 = 2.048$  MHz), which follows the path indicated with the black thin line in figure. The E1 circuit is transported (asynchronous mapping) via the VC12 path indicated with the grey line in figure (1-2-3-4-5-2-5-6). The VC12 path is transported in its turn via VC4 paths indicated with wider grey lines in figure (1-2-3, 3-4-5-2, 5, 5-6)



- a) Compute the justification ratio  $\rho$  (as fraction of justification opportunity bits occupied by dummy bits) in the VC12 in transit from the DXC #5 to the DXC #2 at interface C.

$$2048 \text{ nbits} (1 - 2.5 \cdot 10^{-6}) = (1025 - 2\rho) \cdot 2000 \text{ b/s}$$

$$\rightarrow \rho = 0.50125$$

- b) Compute every how many seconds AU4 pointer justifications do happen at interface C (i.e., the inter-justification period), in the direction from DXC #5 to DXC #2, specifying also their sign (POS/NEG)

$$\text{In the AU4 pointer presence in \#5: } \Delta f = f_{VC4} \left[ -10^{-7} - \left( \frac{2.0479998}{2.048} - 1 \right) \right] =$$

$$1 \text{ just AU4 every } \frac{24 \text{ bit}}{93518 \text{ bit/s}} =$$

$$= 68.13 \text{ sec (POS)}$$

$$= -0.3522 \text{ bit/s}$$

- c) Compute every how many seconds TU12 pointer justifications do happen at interface A (i.e., the inter-justification period), in the direction from DXC #2 to DXC #3, specifying also their sign (POS/NEG)

0 jmt/s

- d) Compute every how many seconds TU12 pointer justifications do happen at interface C (i.e., the inter-justification period), in the direction from DXC #5 to DXC #2, specifying also their sign (POS/NEG)

In the TU12 pointer processor in #3:  $\Delta f = f_{VC12} \cdot 10^{-7} =$

$$1 \text{ jmt TU12 every } \frac{8 \text{ bit}}{9224 \text{ bit/s}} = = +0,224 \text{ bit/s}$$

$$= 35,71 \text{ rc (NEG)}$$

## Problem 2

(Solve on this sheet in the space provided) (6 points)

Let  $s(t)$  be a non-ideal timing signal generated by a clock with instantaneous frequency ( $t \geq 1$  s,  $\nu_0 = 1$  kHz)

$$\nu(t) = \nu_0 + \frac{\nu_0}{t}$$

a) Derive the analytical expression of the timing signal  $s(t) = A \sin \Phi(t)$  knowing that  $\Phi(1) = 0$ .

$$\begin{aligned}\Phi(t) &= 2\pi \int_1^t \nu(\tau) d\tau = 2\pi \nu_0 \left[ t + \log t \right]_1^t = 2\pi \nu_0 (t-1) + 2\pi \nu_0 \log t \\ &= 2\pi \nu_0 (t-1 + \log t)\end{aligned}$$

b) Evaluate the *Time Interval* measured by this clock and its *Time Interval Error* in  $1 \text{ s} \leq t \leq 10 \text{ s}$ , that is  $\text{TI}(t, \tau)$  and  $\text{TIE}(t, \tau)$  for  $t = 1 \text{ s}$ ,  $\tau = 9 \text{ s}$ .

$$T(t) = \frac{\Phi(t)}{2\pi \nu_0} = t - 1 + \log t$$

$$\text{TI}(t, \tau) = T(t+\tau) - T(t) \quad \text{TI}(1, 9) = T(10) - T(1) = T(10) =$$

$$\begin{aligned}\text{TIE}(t, \tau) &= \text{TI}(t, \tau) - \tau = &= 9 + \log 10 = 11,3 \text{ sec} \\ &= 11,3 \text{ sec}\end{aligned}$$

c) On what instant of ideal time  $t$  the *Time Error* of this clock is  $\text{TE}(t) = 0$ ?

$$\text{TE}(t) = T(t) - t = -1 + \log t$$

$$\text{TE}(t) = 0 \text{ per } t = e = 2,7 \text{ sec}$$

**Problem 3**

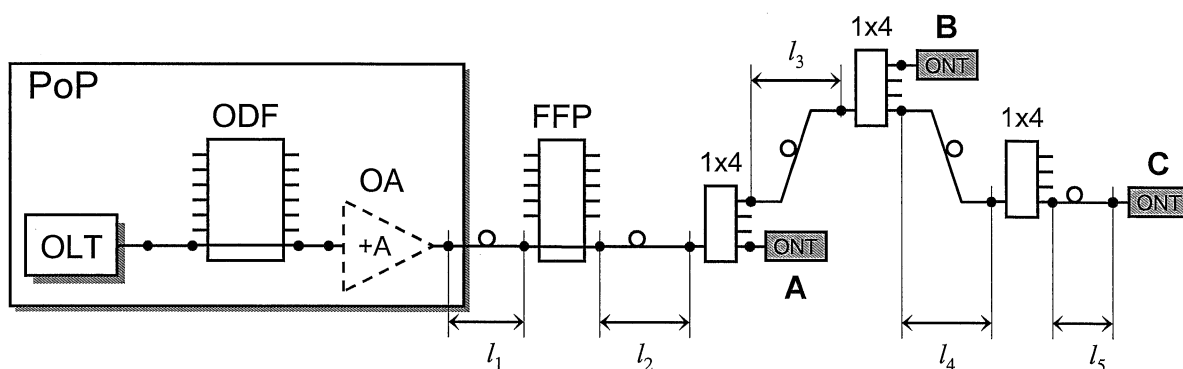
(Solve on this sheet in the space provided) (6 points)

Consider a Passive Optical Network reaching up to 64 users at variable distances from the Optical Line Termination (OLT) via a variable number of 1x4 splitters, with an asymmetric tree topology according to the scheme in figure.

The line from the OLT is cross-connected via an Optical Distribution Frame (ODF) to the PON. An Optical Amplifier (OA), if needed, may be added after the ODF at the Point-of-Presence (PoP). After a first single feeder fibre segment with length  $l_1$ , another ODF (Fibre Flexibility Point, FFP) cross-connects to the PON. The fibre segments between the FFP and the following splitters have length  $l_2, l_3, l_4, l_5$ , respectively. The length of other segments of fibres connecting network elements is negligible. The Optical Network Terminations (ONT) can be connected at the output of any splitter at the three stages (A, B, C).

Assume the following data for the PON elements:

- fibre with attenuation  $\alpha = 0.5$  dB/km;
- $l_1 = 8$  km,  $l_2 = 5$  km,  $l_3 = 2$  km,  $l_4 = 2$  km,  $l_5 = 2$  km;
- OLT transmission power  $P_{TX} = -3$  dBm;
- splitter insertion loss  $\alpha_s = 1$  dB;
- power loss by each couple of optical connectors  $\alpha_c = 0.5$  dB (connections marked with dots in figure);
- sensitivity of ONT receivers  $P_{RX} > -27$  dBm, with at least 6 dB of safety margin to be guaranteed;
- optional OA gain  $+A$  [dB];



- a) Evaluate the maximum *Differential Path Loss* [dB] between ONTs as in figure.

$$DPL = P_{RX/A} - P_{RX/C} = \alpha(l_3 + l_4 + l_5) + \alpha_c \cdot 5 + 2(\alpha_s + 6) \quad [dB]$$

$$= 19.5 \text{ dB}$$

- b) Evaluate the power  $P_{RX}$  [W] received by the farthest ONT in position C without OA.

$$P_{RX/C} = P_{TX} - \alpha_c \cdot 12 - 3(\alpha_s + 6) - \alpha \sum_{i=1}^5 l_i = -39.5 \text{ dBm}$$

$$= 112 \text{ nW}$$

- c) Evaluate the OA gain  $+A$  [dB], which is necessary in order to meet the receiver sensitivity of ONT in position C guaranteeing 6 dB safety margin.

$$A \geq \underset{\substack{\uparrow \\ -2\alpha_c}}{-27 \text{ dBm}} + 6 \text{ dB} - (-39.5 \text{ dBm}) = 18.5 \text{ dB}$$

- d) Evaluate the maximum total distance  $L = \sum_{i=1}^5 l_i$ , which can be covered including an OA with gain  $A = +10$  dB, meeting the receiver sensitivity and safety margin of the ONT.

$$P_{Rx/C} \geq -21 \text{ dBm}$$

$$P_{Tx} + \overset{(-2\alpha_c)}{A} - \alpha_c \cdot 12 - 3(\alpha_s + 6) - \alpha L \geq -21 \text{ dBm}$$

$$-\alpha L \geq -\cancel{0} \text{ dBm} \rightarrow L \leq 0 \text{ Km}$$

Problem 4

(Solve on this sheet in the space provided) (6 points)

Consider a group of  $m = 5$  telephone circuit lines loaded by Pascal Traffic with  $A_0 = 5$  Erl,  $\sigma_0^2 = 7$  (VMR = 1.4). The channel group is modelled as a pure loss queuing system without buffer: a call request is served if at least one line is available, otherwise is rejected and gets lost (i.e., the user gives up, or BCC).

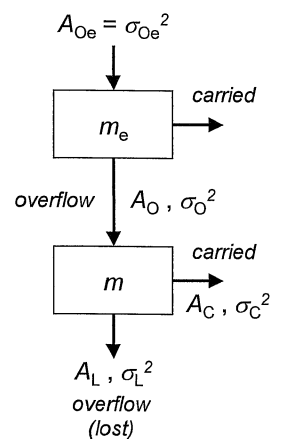
Apply the *Wilkinson's Equivalent Random Traffic Method* and compute:

- the equivalent Poisson traffic  $A_{0e}$  (with  $\sigma_{0e}^2 = A_{0e}$ ) and the number of lines  $m_e$  of the fictitious channel group overflowing (approximately) the given Pascal traffic with  $A_0$ ,  $\sigma_0^2$ ;
- the traffic  $A_L$  [Erl] lost by the group of  $m = 5$  lines loaded by the given Pascal Traffic with  $A_0 = 5$  Erl,  $\sigma_0^2 = 7$ ;
- the traffic  $A'_L$  [Erl] lost by the group of  $m = 5$  lines loaded by a Poisson Traffic with  $A_0 = 5$  Erl and  $\sigma_0^2 = A_0$ .

For you convenience, some key formulas are attached.

Erlang-B Formula:  $E_{1,m}(A_0) = \frac{\frac{A_0^m}{m!}}{\sum_{k=0}^m \frac{A_0^k}{k!}}$   $E_{1,m}(A_0) = B_m(A_0) = \frac{A_0 B_{m-1}(A_0)}{m + A_0 B_{m-1}(A_0)}$

Wilkinson's Overflow Traffic: 
$$\begin{cases} A_0 = A_{0e} E_{1,m_e}(A_{0e}) \\ \sigma_0^2 = A_0 \left[ 1 - A_0 + \frac{A_{0e}}{1 + m_e + A_0 - A_{0e}} \right] \\ A_{0e} = (1 + m_e + A_0) \frac{\sigma_0^2 + A_0^2 - A_0}{\sigma_0^2 + A_0^2} \end{cases}$$



Target:  $A_0 = 5$   $\sigma_0^2 = 7$

$m_e$	$A_{0e}(m_e, A_0, \sigma_0^2)$	$\hat{A}_0(m_e, A_{0e})$	$\Delta = \hat{A}_0 - A_0$
1	5,90625	5,05105	—
2	6,75000	5,03659	—
3	7,59375	5,01962	—
4	8,43750	5,0052	← min
5	9,28125	4,97959	—

$$\Rightarrow \left| \begin{array}{l} A_{oe} = 8,43750 \quad (= G_{oe}^2) \\ m_e = 4 \end{array} \right.$$

$$b) A_L = A_{oe} E_{1, m_e + m}(A_{oe}) = 1,65107 E_{2l}$$

$$E_{1,9}(8,4375) = 0,195682$$

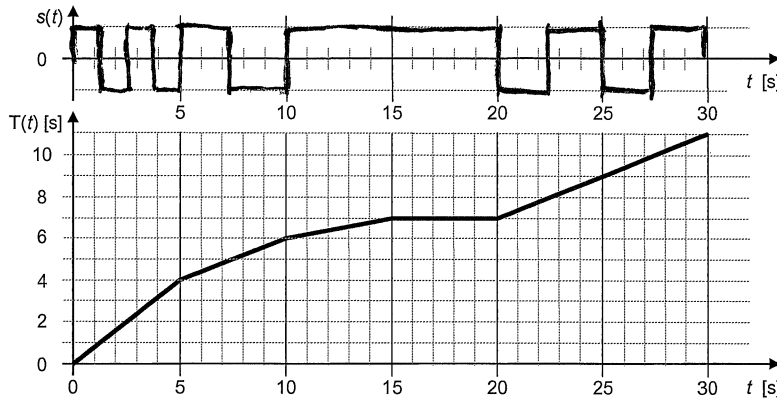
$$c) A'_L = A_o E_{1, m}(A_o) = 1,42434 E_{2l}$$

Problem 5

(Answer on this sheet in the space provided) (12 points)

NB: In any exercise, any answer not justified adequately, even with few words, will not be considered.

- 1) Let  $s(t)$  be a square timing signal with nominal frequency  $\nu_n = 0.5$  Hz and Time  $T(t)$  as plotted in figure. (3 points)
- What is its average frequency [Hz] measured over the ideal time interval  $0 \leq t \leq 30$  s?
  - What is its Time Error  $TE(t)$  at  $t = 30$  s?
  - Plot on the graph the square timing signal  $s(t)$ .



$$\overline{\nu(t)} \Big|_{(0,30)} = \frac{5.5 \text{ cycles}}{30 \text{ sec}} = 0.1833 \text{ Hz}$$

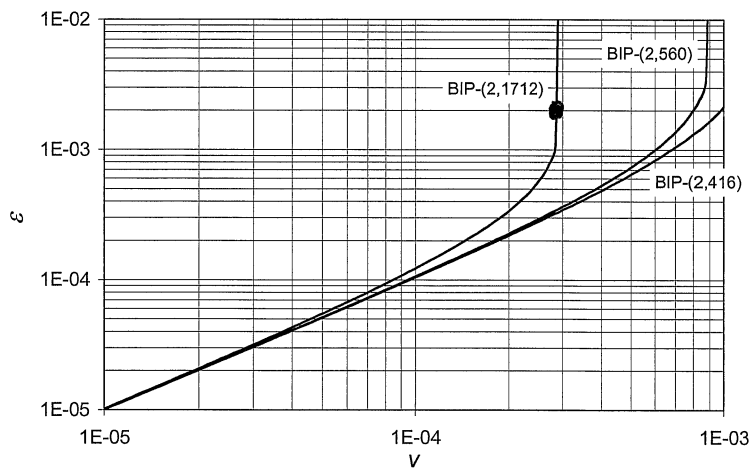
$$\text{or: } \frac{\overline{\nu}}{\nu_n} = \frac{11 \text{ sec}}{30 \text{ sec}} \rightarrow \overline{\nu} = \nu_n \frac{11}{30}$$

$$T(30 \text{ s}) = 11 \text{ s}$$

$$TE(30 \text{ s}) = -19 \text{ s}$$

$$\nu_n = 0.5 \text{ Hz} \rightarrow 1 \text{ cycle in } 2 \text{ sec} \quad T(t) = \frac{\Phi(t)}{2\pi\nu_n}$$

- 2) The figure below plots the calibration curves  $\varepsilon(\nu)$  for adjusting BER estimates of bits 1, 2 of byte V5 of VC-11, VC-12 and VC-2. (2 points)
- Under the error rate  $BER = 0.002$  on the transmission line, what is the approximate parity violation rate detected by the byte V5 of VC-2?
  - What is the maximum rate of random transmission errors? What is the maximum parity violation rate detectable by byte B3 of VC-4 in such a case?



$$E = 0.002 \rightarrow \nu \approx 2.1 \cdot 10^{-4}$$

$$E_{\max} = 0.5$$

$$\nu = \frac{1}{2(m+1)} = 2.1 \cdot 10^{-4}$$

$$m = 261.9 = 2349$$



**Last and first name:***(capital letters)**(signature)*

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- 3) Outline the principle of *Virtual Concatenation* (what is different from Contiguous Concatenation?) enhanced by *Link Capacity Adjustment Scheme*. *(2 points)*

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- 4) What is the Digital Divide problem in access networks? To what is it due?

*(2 points)*

- 5) Outline the principle of *Dynamic Bandwidth Allocation* in Passive Optical Networks, highlighting differences and possible advantages versus static allocation. Who assigns slots? Based on what criteria? (3 points)